(Submitted for publication)

Computer Cartograms

Waldo Tobler

Professor Emeritus

Geography Department

University of California

Santa Barbara, Ca 93106-4060

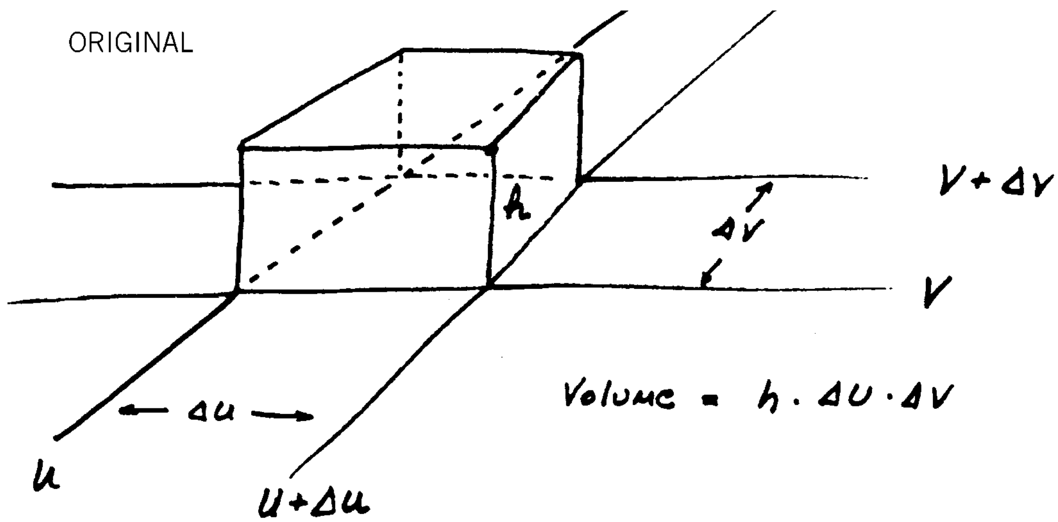
http://www.geog.ucsb.edu/people/tobler.htm

Abstract

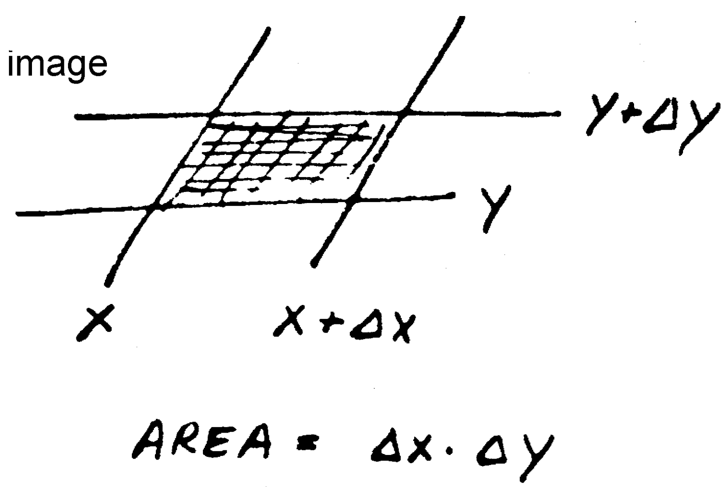
Based on a lecture from the mid 1960's, a direct and simple introduction is given to the design of a computer algorithm for the construction of contiguous value-by-area cartograms. Both the regular and the irregular polygon cases are treated. There then follows a presentation of the subsequent history of the subject including variant algorithms. As an example a table of latitude/longitude to rectangular plane coordinates is included for a cartogram of the United States. The value of treating the subject within the context of map projections is explained.

Keywords: Anamorphoses, cartograms, map distortion, map projections.

A value-by-area cartogram is a map projection that converts a measure of a non-negative distribution on the earth to an area on a map. Consider first a distribution h (u, v) on a plane:



Next consider an area on the map:



We want these to be the same. That is, the map image is to equal the original measure:

Image area on map = Original volume on surface,

Or

$$\Delta x \, \Delta y = h \, \Delta u \, \Delta v.$$

As an aside, observe that in his treatise of 1772 J. H. Lambert defined an equal area map projection in exactly this fashion, setting spherical area equal to map surface area (Tobler 1972).

Now replace the Δ's by d's, i.e.

$$dx \, dy = h \, du \, dv.$$

This can be rewritten in integral form to cover the entire domain as

$$\iint dx \, dy = \iint h \, du \, dv$$

To solve this system we can insert a transformation, i.e., divide both sides by dudv to get dxdy/dudv = h (u, v) which can be recognized as the Jacobian determinant. That is, with this substitution the condition equation becomes

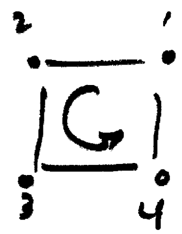
$$J = \partial (x, y)/\partial (u, v) = h (u, v).$$

Written out in full we have the equation

$$\partial x/\partial u \, \partial y/\partial v - \partial x/\partial v \, \partial y/\partial x = h (u,v).$$

To apply this to a sphere it is only necessary to multiply by $R^2 \cos \varphi$ on the right hand side of this equation, substituting longitude (λ) and latitude (φ) for the rectangular coordinates u and v.

Reverting now to the pictures on the plane, a small rectangle will have



nodes identified by cartesian coordinates given in a counterclockwise order:

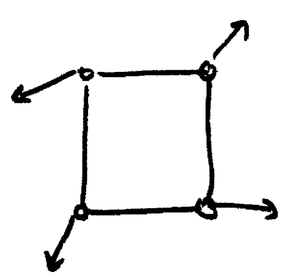
The area of such a rectangle is given by the determinant formula

$$2A = \begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix} + \begin{vmatrix} X_2 & Y_2 \\ X_3 & Y_3 \end{vmatrix} + \begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix} + \begin{vmatrix} X_4 & Y_4 \\ X_1 & Y_1 \end{vmatrix}$$

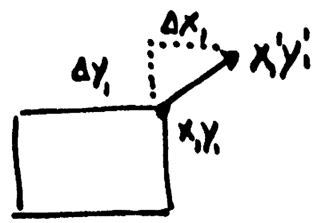
It is now desired that this area be made equal to the "volume" h (u, v). This can be done by adding increments Δx, Δy to each of the coordinates:

The new area can be computed from a formula as above but now using the displaced locations $X_i + \Delta X_i$, $Y_i + \Delta Y_i$, $i = 1, \ldots ,4$.

Call the new area A'. Now use the condition equation to set the two



areas equal to each other A' = A. Recall that A' is, by design, numerically equal to h (u, v). The equation A' = h = A involves eight unknowns, the ΔX_i



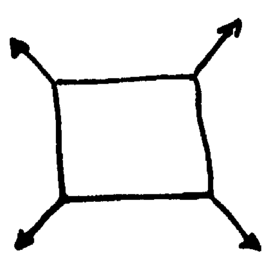
and the ΔY_i , i = 1…4. Now it makes sense to invoke an isotropicity condition, to attempt to retain shapes as nearly as possible. So set all ΔX_i and ΔY_i equal to each other in magnitude, and simply call the resulting value Δ. That is, assume

$$\Delta X_2 = -\Delta X_1 \qquad \Delta Y_2 = \Delta Y_1$$

$$\Delta X_3 = -\Delta X_1 \qquad \Delta Y_3 = -\Delta Y_1$$

$$\Delta X_4 = \Delta X_1 \qquad \Delta Y_4 = -\Delta Y_1$$

and finally that $\Delta = \Delta X_1 = \Delta Y_1$.



This is also the condition that the transformation be, as nearly as possible, conformal, that is, locally shape preserving, and minimizes the Dirichlet integral

$$\int_R (\partial x^2/\partial u + \partial y^2/\partial v + \partial x^2/\partial v + \partial y^2/\partial u)\, dudv.$$

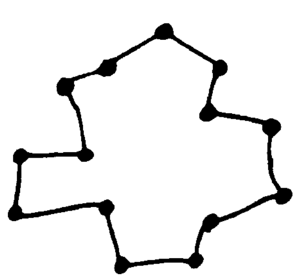
And this renders the transformation unique. Thus A’ is just an enlarged (or shrunken) version of A. Working out the details yields

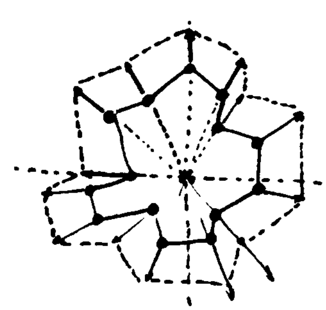
$$A' = 4\Delta^2 + \Delta (X_1 - X_2 - X_3 + X_4 + Y_1 + Y_2 - Y_3 - Y_4) + A$$

And this needs to be solved for the unknown Δ. Once This quantity is found the problem has been solved. The map area A’ is now numerically equal to the numerical value of h (u,v).

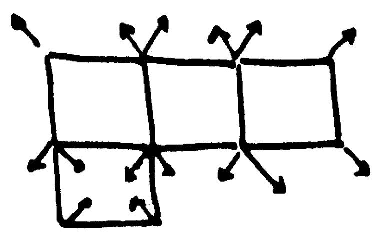
Many different equal area map projections are possible, and the same holds for value-by-area cartograms. There is one defining equation to be satisfied but this is not sufficient to completely specify a map projection. The choice of a transformed map that looks as nearly as possible like the original was made above but there are other possibilities for the second projection condition.

If the regions with which one is dealing are irregular polygons, instead of rectangles, the procedure is exactly the same. One simply translates to the centroid of a polygon and then expands (or shrinks) by the proper amount to get the desired size. The scale change is the square root of (A’/A) , from which Δ is easily calculated.

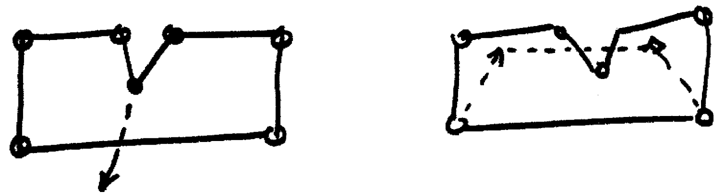




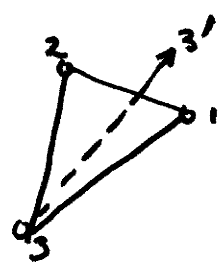
When two rectangles, or two polygons, are attached to each other there will nodes in common. The amounts of displacement calculated independently for such a node will differ. Suppose one displacement is calculated to be Δ_1 and the other Δ_2. Then take the average. But this means that neither will result in the desired displacement. In particular, in a set of connected rectangles, or polygons, this problem will occur at most nodes. Some will be connected to more than two regions. Again, just take the average of all displacements. After all displacements have been calculated apply them by adding the increments to every node. Then repeat the process in a convergent iteration. Eventually all regions will obtain their proper, desired size.



There remains one final problem. The displacement at a node can be such that it requires that the node cross over the boundary of some region. This must be prevented. Equally seriously displacing two nodes may result in the link between them crossing over some other node. And this again must be prevented. Under-relaxation - shrinking the displacements to some fraction,

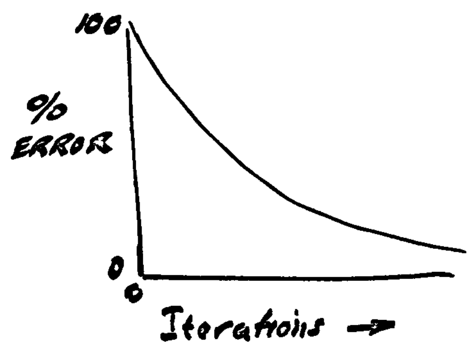


say 75%, of the desired values helps avoid, but does not prevent, the problem. The technique used in the computer to solve this difficulty is to recognize that when a node of a triangle crosses over the opposite edge of the triangle then the area changes sign, from positive to negative. Every node must be checked



against every boundary link, and every link must be compared to every node. If a crossing is detected then shrink the displacement until it is no longer is a problem. This topological checking slows the algorithm down considerably. The next iteration will compute a new displacement and the desired result is eventually achieved. The topological check also prevents negative areas from occurring. Negative areas are not permitted, since by assumption, h (u, v) is non-negative.

The "error", the discrepancy between the desired result and the result obtained, is measured by $\sum |A - A'| / \sum A$ with A' normalized so that $\sum A' = \sum A$ over all areas. The convergence of the algorithm then follows a typical exponential decay. My experience has been on an IBM 709 computer with data given



by latitude and longitude quadrilaterals. Or using states as the data-containing polygons. Using one degree population data for the continental United States, with a 25 by 58 lattice (= 1450 cells) the program required 25 seconds per iteration.

Postscript

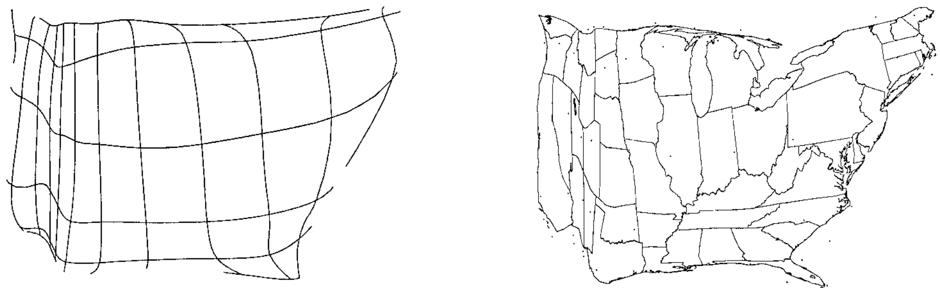
The foregoing is a transcription of a lecture delivered in the mid 1960's to Howard Fischer's computer graphics group at Harvard University. Two computer versions then existed. These were based on my 1961 Ph.D. thesis.

One program treats data given by latitude/longitude quadrangles, the other is for irregular polygons. The lattice version worked on the latitude/longitude grid (or any orthogonal lattice) and produced a table of x, y coordinates for each degree of latitude and longitude. In other words the program produced tables of $x = f(\varphi, \lambda)$, $y = g(\varphi, \lambda)$. These are the two equations required to generate a map projection. An example is given here in the table.

Map projectioon coordinates for a cartogram of the United States
Values in degrees

LATITUDE / LONGITUDE		24 N	29 N	34 N	39 N	44 N	49 N
125 W	X	0.901	0.512	0.258	0.916	1.469	0.705
	Y	0.520	1.295	6.529	17.856	23.559	28.786
120 W	X	2.572	1.744	1.428	3.050	3.350	3.772
	Y	0.272	1.151	7.256	16.240	22.415	29.166
115 W	X	5.364	5.634	5.422	4.351	4.779	5.505
	Y	0.384	1.203	7.656	14.756	22.841	27.355
110 W	X	6.457	6.526	6.196	5.171	6.072	6.231
	Y	0.519	1.643	9.624	15.592	23.570	27.464
105 W	X	7.124	7.118	6.897	6.082	6.783	7.151
	Y	0.526	1.908	10.340	16.850	23.989	27.529
100 W	X	8.375	8.474	8.035	7.875	8.102	7.985
	Y	0.177	1.526	9.030	17.452	23.991	27.597
95 W	X	12.221	13.372	10.826	10.653	9.944	10.867
	Y	0.205	1.873	9.683	17.012	24.102	28.341
90 W	X	16.575	16.741	17.082	14.738	15.994	17.012
	Y	0.481	1.492	8.323	16.218	25.606	28.773
85 W	X	23.867	24.124	23.866	24.013	24.001	24.279
	Y	0.325	1.401	6.476	14.864	26.785	28.610
80 W	X	33.333	33.993	32.520	32.508	33.131	32.813
	Y	0.618	2.031	4.538	13.440	26.556	28.511
75 W	X	41.886	42.043	43.210	40.877	39.962	39.961
	Y	2.195	2.971	4.184	8.479	26.184	28.638
70 W	X	46.932	46.854	47.618	48.131	48.249	47.535
	Y	2.085	2.958	5.590	10.501	26.452	29.172
65 W	X	48.652	48.117	48.811	49.469	49.630	49.291
	Y	1.311	2.718	6.350	13.588	25.608	28.695

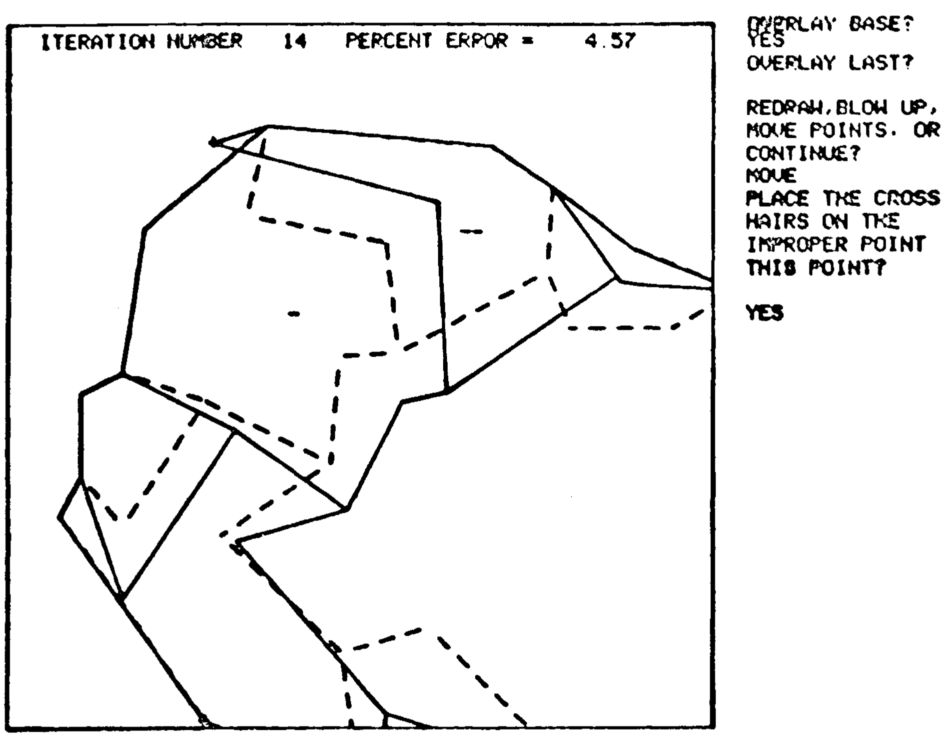
Using a standard map projection program it is possible to plot coastlines, state boundaries, rivers, etc., by interpolation using data given in these coordinates. At that time the University of Michigan CalComp plotter was used. The figure shows an example (not from the coordinate table above – based on a different set of data).



A separate program, again independent of any particular map projection, computed the linear and angular distortion of Tissot's (1881) indicatrix by finite differences from this same coordinate file. Tissot's measure of area distortion on map projections had been shown (Tobler 1961) to be equal to the desired area on a cartogram. The cartogram program was tested by entering the spherical surface area for latitude/longitude quadrangles and, as expected, yielded an equal-area map projection for the United States similar to that by Albers. The second program used state outlines, or general polygons instead of a grid, directly. Both programs allowed one to specify particular points to be fixed – not to be moved – so that, for example, some exterior boundaries could be fixed. Or points critical for recognition of landmarks could be retained. Both programs also had an option to produce an initial pseudo-cartogram, treating the quantity to be preserved, h (u, v), as if it could be approximated by a separable function of the form h_1 (u) h_2 (v). To do this the program "integrates" the data in the u direction and then separately in the v direction. The effect is somewhat like using a rolling pin in two orthogonal directions on a batch of bread dough. This is used to compute a beginning configuration from which to initiate the iterations, saving computer time, but it also affected the resulting appearance of the cartogram. A similar residual effect can be observed if a cartogram is begun with information given on a world map projection such as the sinusoidal or Mollweide. Another trick is to begin with a simplified set of polygon outlines, iterate to convergence, and then restart with more detailed polygon outlines, etc. Since data given by political units (polygons) often varies dramatically from one polygon to the next it also makes sense to use pycnophylactic reallocation, which does not change the total data within any polygon, but redistributes it to obtain a smoother arrangement with less drastic fluctuation from polygon to polygon. If using finite differences (Tobler 1979) for this smooth reallocation, the individual polygons are partitioned into small quadrangular cells and the computer version for a regular lattice is used, and the coordinates of the cell vertices are retained for subsequent plotting. If finite elements with triangles (Rase 2002) are used to implement the smoothing reallocation within the polygons then the computer program version for irregular areas must be used.

In addition to the Harvard lecture the algorithm was also presented to a conference on political districting (Tobler 1972) and at a conference on computer cartography applied to medical problems (Tobler 1979). A description of how to proceed when the geographical arrangement of phenomena is known through the use of an approximating mathematical equation was also given in the 1961 thesis, in the 1963 paper, in the 1979 paper, and in the 1974 program documentation. Additional programs at that time could produce a hexagonal grid to cover a region and to produce an inverse transformation from the lat/lon grid and the tables of x, y coordinates. The tables then are for $\varphi = f^{-1}(x, y)$ and $\lambda = g^{-1}(x, y)$. This allows the inverse transformation of the hexagonal grid to lie, in warped form, over the original image, in an attempt to replicate Christaller's central place theory in a domain of variable population. An example is shown in the 1972 paper, and reprinted in Tobler (1976).

An interactive version of the polygon program version, using a Tektronix display terminal, was prepared in 1970 by Stephen Guptill and myself, and presented at a computer conference in 1974. This program allowed the topological checking to be performed visually, thus avoiding the need for the tedious computer topological solution. The discrepancy in each polygon was indicated by scaled plus or minus symbols at the polygon centroids and the proposed change at each iteration could be superimposed on the previous result, or on the original configuration, shown as a set of dashed lines. Using the interactive cursor the cartographer could zoom in on a location on the map to move and improve offending, or inelegant, displacements. The computer costs were then reduced 100-fold since the topological checking could be disengaged. An example is shown in the figure for a portion of a map of South America.



Most of the FORTRAN programs were distributed in a 110 page Cartographic Laboratory Report (No. 3, 1974) from the Geography Department at the University of Michigan. Several of these programs were later (circa 1978) implemented on a Tektronix 4054 in the Geography Department at the Santa Barbara campus of the University of California.

In 1971 G. Ruston published a computer program that uses a physical analogy. One may imagine that a thin sheet of rubber is covered with an uneven distribution of inked dots. The objective is to stretch the rubber as uniformly as possible until the dots are evenly distributed on the sheet. This simple description is an approximate representation of the mathematical statement used for his computer program. If the dots represent the distribution of, say, population, the resulting cartogram is such that map areas are proportional to the population. Hexagons drawn on this surface can represent market areas when the rubber sheet is relaxed to its original, pre-stretched, form. Uniqueness would seem to me to depend on boundary conditions.

In 1975 A. Sen published a theorem about cartograms. In effect he asserted that the least distorted cartogram has the minimal total external boundary length of all possible areal cartograms. Interestingly this had been empirically discovered by Skoda and Robertson in 1972 while constructing a physical cartogram of Canada using small ball bearings to represent the unit quantity. Similar methods were reported by Hunter & McDowell (1968) and by Eastman, Nelson & Shields (1981).

Kadman and Shlomi in 1978 introduced the idea that a map could be expanded locally to give emphasis to a particular area. Hägerstrand (1957) already did this of course, and Snyder's (1987) "Magnifying Glass" map projection uses a similar idea. More recent versions of this idea appear in Rase (1997, 2001), Sarkar (1994), and in Yang, Snyder, and Tobler (2000).

In 1983 Appel, et al, of IBM patented a cartogram program that worked somewhat like a cellular automaton. Areas were represented as cells and "grew" by changing state (color) depending on the need for enlargement or contraction.

In 1985 Dougenik, Chrisman, and Niemeyer of the Howard Fischer's Harvard Computer Graphics Laboratory published an algorithm that differed from the one that I developed in a small but important respect. Instead of applying the displacements to all nodes after all of them had been computed, as in my algorithm, they applied them immediately to all nodes. This was done after computing the required displacements for only one polygon. Displacements, discounted by a spatially decreasing function away from the centroid of the polygon in question, were then applied to all polygon nodes simultaneously. They then moved on to the next polygon and repeated the procedure. Iterations were still required. The result is a continuous transformation of a continuous transformation, and that of course is continuous. As a consequence they avoided the necessity for a tedious topological constraint and virtually all topological problems were avoided. Depending on the complexity of the polygon shapes occasional overlapping might still occur, but only infrequently. Almost all subsequently developed programs stem from this publication.

Three further examples are worth citing. One was presented in 1993 by the Soviet authors Gusein and Tikunov (also Tikunov 1988). Another, with emphasis on medical statistics, formed the basis for a Ph.D. dissertation at the University of California at Berkeley by D. Merrill et al (1991), also Selvin et al (1988). An earlier health oriented paper is by Levinson & Haddon (1965). Most recently A. Herzog of the Geography Department of the University of Zürich has prepared a program for interactive use on the Internet.

D. Dorling, in 1991, developed a novel approach. Using only centroids of areas he converts each polygonal area into a small bubble – a two dimensional circle. These bubbles are then allowed to expand, or contract, to attain the appropriate areal extent. At the same time they attempt to remain in contact with their actual neighbors. Dorling also colors the resulting circles depending on some additional attribute. His work contains, as of the 1991 date, the most comprehensive bibliography on the subject of area cartograms. The bubble algorithm is most popular in the United Kingdom, perhaps because a complete computer program was published, and now also seems to be available on the Internet.

The most recent implementation is from Texas, again in a thesis (Kocmoud, 1997, also Kocmoud & House 1998). This interesting algorithm attempts to maintain shape in addition to proceeding to correct areal sizes. The shape preservation alternates with area adjustment in each iteration. As implemented it is rather slow, but can probably be improved. Other computer scientists (Edelsbrunner &, Waupotisch 1995) have also studied the problem. An undergraduate thesis has also been presented by Inglis (2001), and there is a master's thesis by Torguson (1990). Some literature exists in German (Elsasser 1970), Rase (2001) too.

It has also been suggested that cartograms are difficult to use, although Griffin (1980) does not find this to be the case. Nevertheless Fotheringham, et al (2000, p. 26), in considering Dorling's maps, state that cartograms

"*…can be hard to interpret without additional information to help the user locate towns and cities*".

The difficulty here is that many people approach cartograms as just a clever, unusual, display graphic rather than as a map projection, and not similar in purpose to Mercator's projection as an analogue method of solving a problem. Mercator's map is not designed for visualization, and should not be used as such. If the anamorphic cartograms are approached as map projections then it is easy to insert additional map detail. In the case of Dougenik et al's, or Dorling's, or other, versions simply knowing the latitudes and longitudes of the nodes or centroids allows one to draw in the geographic graticule, or to plot any additional data given by geographic coordinates. And this can be done using a standard map projection program augmented by a subroutine to calculate from a map projection given as a table of coordinates. Included here, in addition to the geographic graticule, could be roads, rivers lakes, etc., and these would enhance recognition. Dorling's bubbles could be replaced by administrative units or other features. One could then even leave off of the map the administrative or political units that were used in the construction of the cartogram. And one could replace these by superimposing an alternate set of boundaries or other information (for example, disease incidence, poverty rates, or shaded topography). This requires a bit of simple interpolation from the known point locations. Another advantage is that a light smoothing can also be applied to the graticule (coordinates) to improve the map's appearance. It is then also possible to use Tissot's results to calculate the angular and linear distortion of the map - the areal distortion, Tissot's S, has been shown (Tobler 1963) to be equal to the distribution being presented. Tissot's indicatrix is useful in comparing two cartograms obtained from the same data using alternative algorithms. A similar result holds true of cartograms that distort using time or cost distance rather than areal exaggeration. It is now also obvious that one can calculate a cartogram to fit on a globe - a mapping of the sphere onto itself. Examples might be to represent surface temperature or annual precipitation. From this any equal area map projection (Albers, Mollweide, sinusoidal, Lambert, etc.) can be used to represent the information as an anamorphose on a flat map.

A few references:

Appel, A., Evangelisti, C., Stein, A., 1983, "Animating Quantitative Maps with Cellular Automata", *IBM Technical Discovery Bulletin* 26, No.3A: 953-956.

Dorling, D., 1991, The Visualization of Spatial Srtructure, Thesis, Department of Geography, University of Newcastle upon Tyne, UK

Dorling, Daniel (1996): "Area Cartograms: Their Use and Creation", *Concepts and Techniques in Modern Geography* (CATMOG), No. 59, Dougenik, James, A., Nicholas R. Chrisman, and Duane R. Niemeyer (1985): ‘An Algorithm to Construct Continuous Area Cartograms”, *Professional Geographer*, 37 (1), 75-81.

Du, Changming, 1999, “Constructing Contiguous Area Cartogram using ArcView Avenue”, ESRI International Conference, *Paper No. 489*, San Diego, CA

Eastman, J, Nelson, W, & Shields, G, 1981, “Production considerations in isodensity mapping”, Cartographia, 18(1):24-30.

Edelsbrunner, H., Waupotisch, R., (1995) “A combinatorial approach to cartograms”. *Proceedings* of the 11th Annual Symposium on Computational Geometry. ACM Press, 98-108

Elsasser, H., 1970, “Nichtflachenproportionale Kartogramartige Darstellungen der Schweiz” *Geographica Helvetica*, 25, 2: 78-82.

Fotheringham, A., Brunsdon, C., Charlton, M., 2000, *Quantitative Geography*, Sage, London.

Griffin, T., 1980, “Cartographic Transformation of the Thematic Map Base”, *Cartography*, 11, 3 (March): 163-174.

Gusein-Zade, S., Tikunov, V., (1993), “A new method for constructing continuous cartograms”, *Cartography and Geographic Information Systems*, 20, 3: 167-173

Hägerstrand, T. (1957) “Migration and area”. In: *Migration in Sweden*, Lund Studies in Geography, Ser. B, Human Geography, No. 13, 1957, 27-158

Hunter, J., and Young, J., 1968, “A Technique for the Construction of Quantitative Cartograms by Physical Accretion Models”, *Professional Geographer*, 20, 6: 402-407.

Inglis, K., 2001, “Computer Generated Cartograms”, undergraduate thesis, Computer Science, Emmanuel College, UK.

Jackel, C., 1997, “Using ArcView to Create Contiguous and Noncontiguous Area Cartograms”, CAGIS, 24(2): 101-109

Kadman, N., and Shlomi, E., 1978, “ A Polyfocal Projection for Statistic Surfaces”, *The Cartographic Journal*, 15, 1 (June): 36-41.

Kocmoud, C., (1997) “Constructing continuous cartograms: a constraint-based approach”. MA thesis, Texas A&M University

Kocmoud, C., House, D., (1998) “A constraint-based approach to constructing continuous cartograms”. *Proceedings* 8th International Symposium on Spatial Data Handling, Vancouver 1998

Levison, M., and Haddon, W., 1965, “The Area Adjusted Map: An Epidemiological Device”, *Public Health Reports*, 80, 1 (Jan): 55—59.

Merrill, D., Selvin, S. Mohr, M., (1991) “Analyzing geographic clustered response”, *Proceedings* of the 1991 Joint Statistical Meeting of the American Statistical Association. Also Lawrence Berkeley Laboratory Report No. 30954, June 1991. Also see the references at URL: http://www-library.lbl.gov/docs/LBNL/416/24/HTML/refs.html

Olson, J., 1976, “Noncontiguous Area Cartograms”, *The Professional Geographer*, 28, 4: 371—380.

Rase, W., (1997) „Fischauge-Projektionen als kartographische Lupen“. *Salzburger Geographische Materialien*, Heft 25, Salzburg 1997, 115-122

Rase, W., 2001, „Kartographische Anamorphosen und andere nicht lineare Darstellungen“, *Kartographische Bausteine*, 19, T. U. Dresden.

Rase, W., 2002, „Volume preserving interpolation of a smooth surface from polygon-related data“, *J. Geographic Systems*, 3(2): 199-219.

Ruston, G., 1971, “Map Transformations of Point Patterns: Central Place Patterns in Areas of Variable Population Density”, *Papers and Proceedings*, Regional Science Assn., 28: 111-129.

Sarkar, M.., Brown, M.., (1994) “Graphical fisheye views”. *Communications* of the ACM, Vol. 37, No. 12, December 1994, 73-84

Selvin, S., Merrill, D., Schulman, J., Sacks, S., Bedell, L., Wong, L., 1988, "Transformations of Maps to Investigate Clusters of Disease", Soc. Sci. Med., 26, 2: 215-221.

Sen, A., 1967, "Uniformizing Mappings", unpublished manuscript, Toronto.

Sen, A., 1976, "On a Class of Map Transformations", *Geographical Analysis,* 8, 1: 23-37.

Sen, A., 1975, "A Theorem Related to Cartograms", *American Mathematical Monthly*, 82, 4: 382-385.

Skoda, L., and Robertson, J., 1972, "Isodemographic Map of Canada", *Geographical Paper No. 50*, Department of Environment (En 36-506/50), Ottawa.

Snyder, J., 1987, "Magnifying Glass Azimuthal map Projections", *Am. Cartographer*, 14, 1: 61-68.

Tikunov, V., 1988, "Anamorphated cartographic images: historical outline and construction techniques”, Cartography, 17, 1: 1-8

Tobler, W, 1961, “Map transformations of Geographic Space”, Ph.D.Thesis, University of Washington, Seattle (U. Microfilm #61-4011).

Tobler, W., 1963, “Geographic Area and Map Projections”, *Geographical Review,* 53: 59-78.

Tobler, W., 1972 "Lambert's Notes on Maps, 1772", Introduced and translated, Michigan Geographical Publication No. 8, Ann Arbor, 125pp.

Tobler, W., 1973, “A Continuous Transformation Useful for Districting”, *Annals*, New York Academy of Science, 119: 215-220.

Tobler, W., 1974, “Cartogram Programs for Geographic data given by Latitude - Longitude Quadrilaterals, by Irregularly Shaped Polygonal Areas, and by a Mathematical Equation”, *Cartographic Laboratory Report* No. 3, Ann Arbor, 110 pp.

Tobler, W., 1976 "Analytical Cartography", *The American Cartographer*, 3 (1976): 21-31. .

Tobler, W., 1979, “Cartograms and Cartosplines”, pp. 53-58 of *Automated Cartography and Epidemiology*, Department of Health, Education, and Welfare PHS-79-l254, Washington DC.

Tobler, W., 1979, “Smooth pycnophylactic interpolation for geographic regions,” *Journal of the Am. Statistical Association*, 74, 367 (Sept): 519-536.

Tobler, W., 1984, “Interactive Construction of Contiguous Cartograms, National Computer Graphics Association Conference, *Computer Graphics ’84*, Anaheim, CA, May 13-17.

Tobler, W., 1986 "Pseudo-Cartograms", *The American Cartographer*, 13,1, 43-50.

Torguson, J., 1990, “Cartogram: A Microcomputer Program for Interactive Construction of Contiguous Value-by-Area Cartograms”, MA thesis, University of Georgia, Athens. 108pp.

Tissot, M., 1881, “Mémoire sur la representation des surfaces et les projections des cartes géographiques”, Herman, Paris, 206 pp.

Yang, Q., Snyder, J., Tobler, W., 2000, *Map Projection Transformation: Principles and Applications*, Taylor & Francis, London. 367 pp.