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The selection of class intervals

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MS received 23 June 1976

ABSTRACT. The selection of class intervals, which can strongly affect the visual impression given by a map, is currently a totally anarchic branch of cartography. While practising cartographers have barely accustomed themselves to the routine techniques of class selection, recent work has widened the choice available and extended the opportunity to produce desired bias.

Systems of class intervals, apart from those fixed exogenously or in arbitrary fashion, are classified into idiographic or serial types, the latter being recommended here. Scale transformations leading towards symmetrical frequency distributions are important and are required for proportional as well as graded symbolization. It is suggested that class intervals should not be optimized in relation to details of the statistical frequency distribution, but should be selected according to the overall shape of this distribution. For rectangular distributions, equal division of the range is appropriate: for dominantly unimodal distributions, intervals related to the standard deviation (on a scale which makes the distribution symmetrical) and for J-shaped distributions, geometric progressions to bases which are greater as skewness increases. Techniques are given for calibrating geometric progressions relative to the median, and for dealing with the special characteristics of percentages.

> An analysis of maps prepared by authors in various academic disciplines fails to show any rational or standardized procedures for the selection of class intervals. Evidently intuition, inspiration, revelation, mystical hunches, prejudices, legerdemain and predetermined ideas of what the class intervals should be have characterized the work of most map-makers. . . . Apparently many authors believe that maps are an art-form which allow liberties not admissible in verbal or tabular presentation. (Jenks and Coulson, 1963, p. 120)

TO GROUP OR NOT TO GROUP?

THE impression conveyed by a map is a function not only of reality, but also of data quality, definition of variables mapped, class intervals or grouping of data, map scale, graphic design and map perception by the reader. The effects of most of these factors are poorly understood, but the present article concentrates upon the selection of class intervals, a topic on which there is a singular lack of agreement among cartographers. In the first place, although most cartographers treat quantitative data by grouping them into a small number of classes, some 'heretics' such as Tobler (1973) do not even agree that such grouping is desirable.

The resulting debate takes us to the heart of the philosophy of cartography, a subject in which philosophy is implicit more often than it is explicit. Dobson (1973), in an attempt to prevent the spread of Tobler's heresy, provided a useful summary of the rationale of grouping. This starts from the fact that in absolute terms the human eye can confidently identify only a small number of shadings or sizes of symbol. How small this number is must depend upon the constraints on range of symbols available, upon the spatial context of the symbols and the smoothness of variability portrayed, and upon human factors. Generally it is taken as seven or eight (Jenks and Knos, 1961); certainly it would rarely exceed ten or fall below four. This is but one manifestation of the phenomenon of 'channel capacity' in the perception of a single univariate stimulus (Miller, 1956). Hunt (1968) used a series of eight very distinct shadings, and by varying pattern as well as darkness Geddes (1942) presented twelve shadings which are distinct, yet in unambiguous sequence.

Clearly, then, it *may* be desirable to limit the number of classes to that number which users can confidently discriminate. Dobson assumed that this must *always* be desirable, because he made a further, unstated assumption that every map should permit its readers to identify exactly the class to which each individual symbol belongs. Dobson's views on the desirability of limiting the number of classes were firmly supported by Monmonier (1973), and similar views, which were expressed by Robinson and Sale (1969) and by Jenks and Caspall (1971), appear to be those of the majority of cartographers. Yet looking at each symbol individually and comparing it with the legend implies usage of the map principally as a data storage medium, as a 'spatial table'; if this is the aim, it is much more efficient to write the actual numbers on the face of the map (Dickinson, 1973). As Dobson (1974, p. 46) himself stated in another context, 'The cartographer does not expect the dot-map viewer to count dots (Heaven forbid), but to see general tendencies.'

More important map uses (see Bertin's (1975) 'maps to be seen' as opposed to 'maps to be read') include (i) perception of the overall spatial pattern of a map; (ii) comparing patterns of different maps; and (iii) assessing contrasts between adjacent places. For the latter, in particular, the psychological theory for absolute levels is inappropriate. Instead, we should consider the 'just perceptible differences' between contiguous stimuli; in this case, the number of relative levels which can be discriminated is much greater (Haber and Hershenson, 1973, p. 106). It is unlikely that comparing individual symbols with the legend is an efficient method for any of these aims, or that it is the method actually used by the map reader. Hence we should perhaps give more weight to the general fidelity of symbolization, rather than to identifying individual symbol classes. In any case, a legend for well-designed symbolism should be almost redundant: like the state in Marx's theoretical view, it should wither away.

If a map is intended to convey information on spatial structures, on situations rather than on sites, why should it not have the near-infinite gradations of a photograph, as Tobler (1973) suggested? Dobson (1973) did not reply to this point, but presumably he felt that simplification and highlighting are necessary. In other words, we have a replay of the orthophotomaps versus symbolized maps debate (Radlinski, 1968; Hill, 1974). The conclusion of that debate seems to be that while precise classification, symbolization and emphasis are useful for some features, such as roads, the infinite gradations of the photograph should be retained for complex areas of vegetation and different densities of built-up area.

Graded symbolism is most clearly appropriate where the distribution to be mapped falls into 'natural' classes separated by clear breaks, and less so where there is a gradation from one level to another. It is important, then, to establish whether the statistical frequency distribution is continuous or significantly multimodal. In most cases (e.g. Pringle, 1976) it is continuous. Therefore, the grouping of data into a small number of classes is undesirable except in the rare cases where it is important for the reader to discern reliably into which class each individual symbol falls. Reducing the number of classes achieves simplification at the expense of loss of useful detail, especially local contrasts. Generalization of a map in this way is crude and uneven compared with weighted spatial smoothing (Tobler, 1969); it increases grouping error and distorts differences. On the whole, proportional symbolization should be preferred to graded.

The reason why cartographers have almost always used a small number of classes may, however, have been technical rather than philosophical or perceptual. With manual cartography it is easier to draw, for example, eight graded sizes of circle, so that the compass need be set only eight times, than to reset the compass for every symbol (Dickinson, 1973, p. 45). For line shading, it is easiest manually to use a small number of line spacings coinciding with divisions marked on a ruler. Pre-printed stick-down symbols also come in a small number of sizes.

Tobler (1973) pointed out that in automated cartography this constraint is almost removed, for it is very easy for a computer program to fix a density of shading or a size for each symbol, according to some rule of proportionality. This change is important, but constraints remain because most plotting devices work in fixed increments. For many graph plotters, the increment is 0·01 in. Hence if the smallest square to be drawn has a side of 0·03 in, and the largest, 0·20 in, only eighteen sizes are available. (This is doubled where the increment is 0·005 in, which is now becoming common.) The Dresser laser plotter has 16 intensities of grey available for shading (Rhind, 1974): although adjacent levels cannot usually be discriminated by the human eye, especially on a complex map where the context varies, the number available is sufficiently small that the class-interval system is very important. In fact, even where very many gradations are possible (1024 on the HRD-1 laser plotter), it is still necessary to decide the appropriate scale for proportionality. It is the selection of the appropriate scale which is most important, whatever the number of classes required.

NUMBER OF CLASSES

If we decide not to represent a surface by continuous gradations, we must decide how many classes are to be used and on what criteria class boundaries are to be selected. The common situation involves representing a surface by a series of graded shadings, from light to dark, but similar considerations apply to other types of quantitative symbolization, such as thickness of line or area of square.

Deciding on the number of classes is a subjective stage, suffering from a spill-over of the emotional 'graded versus proportional symbol' debate discussed above. With more than ten classes it is difficult for the reader to be sure to which class a particular symbol belongs, hence one might as well use as many classes as is technically feasible and approach proportional symbolization. With five classes or fewer, class identification should be unambiguous, but the information communicated thereby may be less than an approximate identification on a sixteen-level scale can provide.

Within the range four to ten classes, a decision should be influenced by the intended audience, the technical means available, and the spatial pattern of the distribution. A simple, clear-cut map with four or five classes may be better for an unsophisticated audience, inexperienced at reading graphics. Trained eyes may appreciate the extra information which seven or eight classes portray (Schultz, 1961). Poor mapping tools (e.g. the standard set of line printer characters) or poor reproduction facilities may degrade a complex image, and these constraints suggest use of relatively few classes. A smooth surface with highly positive spatial autocorrelation facilitates sub-division into more classes, each of which occupies a band wide enough for it to be identified and to be followed for some distance. Likewise, in patterns of point or line symbols where nearby symbols are similar in value, nearly similar symbols are likely to be either close to each other, or close to comparable symbols forming a continuous band; this facilitates visual discrimination. Finally, data accuracy should *not* be allowed to influence the decision, for inaccuracy can be used as an argument either for few classes, reducing the chance of being in the wrong class, or for many classes, reducing the portrayed magnitude of error when a measurement does fall in the wrong class.

A CLASSIFICATION OF CLASS-INTERVAL SYSTEMS

So many systems of class intervals have been proposed that it is useful to propose a new two-level classification, which distinguishes four distinct approaches.

1. i EXOGENOUS, fixed in relation to meaningful threshold values relevant to but not derived from the data to be mapped, e.g. a sex ratio of 1 or a critical population density threshold.

2. ii ARBITRARY, usually round numbers of no particular significance. Often the step size is constant in one part of the scale and then changes, e.g. 5, 10, 20, 30, 40, 80, 120, thus completely distorting the underlying 'statistical surface'. Sometimes such intervals may be roughly rounded approximations to true serial progressions (Mackay, 1963).

3. IDIOGRAPHIC, affected by specific details of the data set mapped. This can be subdivided into:

iii Multimodal, separated by 'natural breaks' where frequency is lower.

iv Multi-step, subdivided by 'natural breaks' where the gradient of a cumulated frequency plot changes. This is often erroneously labelled 'clinographic', but the cumulated area or number plot is more analogous to the hypsometric curve, and Clarke (1966) has clearly indicated that this does not portray real surface gradients.

v Contiguity-biased, classified so as to maximize the extent and minimize the number of regions with a given shading class.

vi Correlation-biased, classified so as to maximize similarity to a given map.

vii Percentile (quantile) classes which contain equal numbers of spatial divisions, *or* near-equal areas. ('Quantile' is the more general term, but tends to be confused with 'quartile' and 'quintile'.)

viii Nested-means class limits; a frequency distribution is balanced about its mean, which forms the most obvious point of division to give two classes; each of these classes may be subdivided at its own mean; and so on, giving 2, 4, 8, 16, . . ., 2^m classes (Scripter, 1970).

4. SERIAL, with limits in a definite mathematical relation to each other, fixed in relation to statistics for the overall frequency distribution such as median, mean, and standard deviation or range, but not to individual details of the distribution. Type viii above is marginal to 3. and 4. in that the two-class version fulfils this definition but the higher-order means calculated to permit further sub-division are increasingly affected by details, and the number of fitted parameters is only one less than the number of classes. The other sub-types have class widths which form equal steps on some scale, except for standard deviations which have open-ended highest and lowest classes, and arithmetic progressions which mix equal-interval and progressive concepts and are perhaps open to objection on that basis.

ix Normal percentiles, with class limits that subdivide a normal distribution of appropriate standard deviation into classes equal in frequency (Armstrong, 1969). This is marginal to 3. and 4. in being the only 'serial' system with class limits which do not form a straightforward numerical series. Such classes vary in width in relation to their separation from the mean, but are symmetrical on either side of the mean. This system should not be confused (as in Chang, 1974) with the standard-deviation-based system x, which also works best for a normal distribution. It relates to the percentile system, but is not idiographic since it is calibrated only in relation to mean and standard deviation of the data; its 'percentiles' relate to the theoretical normal frequency distribution. The more the observed distribution differs from the normal, the more unequal will the frequencies of different classes become.

x Standard deviation, with class width defined as a proportion of the standard deviation. A standard deviation is simply the square root of the mean of squared deviations around the mean: it is statistically the most useful measure of dispersion (spread) of a set of measurements. As in ix, class intervals are centred on the mean, which is a class midpoint if the number of classes is odd and a class boundary if the number is even; the highest and lowest classes are necessarily open-ended.

xi Equal arithmetic intervals, with no variation in class width. These may be (a) round numbers, or (b) equal divisions of the arithmetic range.

xii Equal intervals on a reciprocal scale (Jenks and Coulson, 1963). This system cannot be applied if any zero values occur.
xiii Equal intervals on trigonometric scales (sine, cosine, tangent, their reciprocals or their corresponding angles).
xiv Geometric progressions of class width. The *width* of each (higher) class is a constant factor times the width of the preceding class.
xv Arithmetic progressions, with class width increasing by a constant amount in comparison to the next lower class. If this amount is 2 and the first class is 0 to 1, the limits of N classes are 0, 1, 4, 9, 16, 25, . . ., $\sum_{c=1}^{c=N} (1 + 2(c - 1))$.
xvi Curvilinear progressions, where a plot of logarithm of class limit against logarithm of class number forms a smooth curve. Mackay (1963) calibrated these in terms of the upper and lower limits of the lowest class, the lowest limit of the highest class, and the number of classes, and he suggested that the resulting series approximated to current cartographic practice. Class width can either increase or decrease upward, but the rate of change of width decreases upward in all his examples. These series seem to be even more general and flexible than geometric progressions, but they do leave very many free choices to the cartographer, who has little guide to making precise decisions.

It is now possible to consider the relative merits of these sixteen systems, first for a single map, and then for several maps which are to be compared. Measures of the accuracy of class interval systems may then be reviewed.

SELECTING CLASS INTERVALS FOR A SINGLE MAP

Exogenous class limits will be used in the few instances where they are available, but arbitrary limits forming no consistent series are indefensible and should never be used. For most maps, then, it is necessary to choose between the various idiographic and serial alternatives. Most systems make some use of the aspatial frequency distributions of values (but in very different ways); different frequency distributions suggest different class interval systems. It will be seen that there is little relation between the suitability of a system and its current popularity.

Systems iii and iv based on various sorts of 'natural breaks' are widely recommended in the recent literature (Mackay, 1955; Dickinson, 1973, pp. 87–91). They are exemplified by Davis (1974, p. 71) and detailed by Robinson and Sale (1969, p. 169). System iii has been automated by Monmonier (1972, 1973) using principal components, and by Jenks and Caspall (1971) using centroid linkage (see also Dickinson, 1973, fig. 125B). Early proponents of iii included Alexander and Zahorchak (1943), who suggested that minima in the frequency distribution would provide class limits separating clusters of similar areas. They argued that such limits would be significant in themselves, as well as providing simpler spatial boundaries because they pass near fewer of the areas. The force of their argument is however greatly reduced by the difficulty, which they admitted, that a frequency distribution carries no information about spatial pattern. Hence the areas which make a minor peak in the histogram may be scattered over the map, in which case the boundaries based on frequency minima will be no simpler than any others. Jones (1930, p. 181) actually defined his breaks as major steps in the statistical surface, but these steps were high enough to leave plenty of scope for subjective judgement in defining precise class limits for isopleths.

The greatest defect of the 'natural breaks' approach lies in attributing 'significance' to very minor troughs in the histogram. Usually—as in Alexander and Zahorchak's example of county population density—these are just the minor oscillations to be expected in any finely divided histogram. They are unlikely to be replicated either in different regions or at different times, hence they are 'significant' neither in the statistical nor in the everyday sense of the word. If a distribution were demonstrated to be significantly multimodal, that is to consist of a mixture of samples from qualitatively different populations, this approach would be justified, but I know of no cartographic example where this has been demonstrated. Jenks and Coulson (1963, p. 125) found that 'more often than not, clear natural break classes do not occur, and subjective judgements based on frequency graphs vary greatly from cartographer to cartographer'. It is usually possible to find apparent breaks, but these are often the result of small sample size and their significance should not be exaggerated.

System iv involves pairs of contours on either side of minima or maxima in the frequency distribution, or at breaks in cumulated frequency curves. This supposedly divides the cumulated distribution into a series of 'treads' and 'risers'. Again, breaks in the frequency distribution are emphasized, but with the added disadvantage that twice as many contours, or levels of symbolization, are required. In theory, this 'doubling up' of the class boundaries emphasizes the stepped nature of the frequency distribution—*if* the spatial distribution is also stepped. Neither of these two ultra-idiographic approaches, iii and iv, should ever be used in quantitative mapping, unless significant multimodality has been demonstrated statistically.

The scope for increasing the contiguity effect in shading (system v) is limited unless regional classes with overlapping statistical intervals are admitted. It seems better not to distort the map in this way, but to let the degree of contiguity speak for itself. Monmonier's (1972) pioneer results gave very little improvement in contiguity, at the expense of considerable declines in intra-class homogeneity. An alternative and better way of achieving a simpler spatial pattern is to apply weighted spatial averaging (Tobler, 1969). Monmonier's more recent innovation (1975) of automating the selection of class intervals so as to maximize the apparent correlation between maps (system vi) is a more dangerous and indeed sinister weapon to place in the hands of geographers and politicians who know what results they want.

Percentile systems of subdividing frequency distributions are very useful in ensuring equal representation for each class. Of course, if the spatial divisions which provide the units of the histograms vary much in area, as in Hunt (1968), the actual areas shaded, and hence their visual impacts, will vary considerably from class to class. This can be avoided if the histogram is weighted by area (Mackay, 1955) and percentiles are chosen which, as closely as the size of spatial divisions allows, provide an equal map area in each class. When percentiles of data points are chosen prior to isopleth or isarithm mapping by interpolation (e.g. with SYMAP), the areas within each class often vary considerably. In this case, equality of class areas can be achieved by iteration.

It can be argued that an areal percentile map makes maximum use of the number of distinct symbols available. On the other hand, the map reader may wish to compare maps not on the basis of equal divisions of area or of numbers of the irregular divisions for which data are available, but perhaps of equal divisions of the population.

Percentile-based classes are independent of monotonic transformations (e.g. logarithmic or reciprocal, where applicable) of the measurement scale and so they may be preferred when we are unsure of the quantitative basis of the scale (*not* of the accuracy of its measurement, but of its status as an interval or ratio scale). Their main disadvantage is that class intervals vary irregularly in different parts of the measurement scale and between maps of the same variable for different areas or times. This degrades ratio or interval scales to ordinal scales, and hinders comparison between areas or between times. Furthermore, a percentile-based map provides no information whatsoever about the frequency distribution: this could equally well be peaked, rectangular or bimodal. Hence it becomes absolutely incumbent on the cartographer to present a histogram alongside such a map, and to mark the class boundaries upon it. This removes the itter objection and facilitates interpretation in relation to the measurement scale, but it does othing to aid map comparison. Furthermore, percentile classes necessitate very careful reading f the map legend (Schultz, 1961).

Unlike systems ii to vi, percentile classes are replicable between cartographers. Scripter's 1970) recent innovation of the nested-mean system viii is equally objective, and more attractive 1 that it balances the desirable property of equal numbers per class against another desirable roperty, that of equal class widths. Since means minimize second moments (sums of squared eviations), they are the balancing points of the part of the scale which they subdivide, with espect to both magnitude and frequency. Class intervals thus defined are narrow in the modal arts of a frequency distribution and broad in the tails. Extreme values are not allowed to ominate, but they do influence the positions of means of various orders so that the less closely paced the values in a given magnitude range, the broader the classes. For a rectangular frequency istribution, nested means approximate the equal-interval or percentile solutions; for a normal ne, they approximate a standard deviation basis; and for a J-shaped, a geometric progression. Hence nested means provide the most robust, generally applicable, replicable yet inflexible class nterval system. Compared with the three systems just mentioned, nested means have the dis-dvantage of not forming a numerical series independent of the data, and not permitting numbers f classes other than 2^m.

Few examples are as yet available of Armstrong's (1969) standardized class intervals (system x), based on percentiles of the normal probability distribution. For data which follow this istribution, normal percentiles are preferable to true percentiles in being uninfluenced by minor etails of the frequency distribution. Hence they escape the label 'idiographic', although the umerical series which they form is a complex one. Like the standard deviation system, this ystem is unsuitable for markedly non-normal frequency distributions, which it makes apparent hrough unequal class frequencies. Unlike Scripter, Armstrong failed to mention the desirability f transforming the measurement scale to give a more nearly normal distribution (Evans, Catterall and Rhind, 1975), but such transformation is much more necessary here and for system than for nested means.

The particular examples mapped by Armstrong (1969) are mortality ratios, for which there s no theoretical expectation of a normal distribution. His figures 4 and 5 confuse the issue in eing 'light' and 'dark' respectively, not because of positive and negative skewness, but because hey are based on an overall, weighted mean rate, rather than the more appropriate mean of the atios mapped. The standard deviation of these ratios is used, and clearly the frequency distribu-ion of the same ratios is the relevant one for such a map (on which the insignificant ratios should not be mapped; Choynowski, 1959).

For frequency distributions which are approximately normal, or fairly symmetrical with a ronounced mode near the mean, the standard-deviation system x is best. It has the edge over ystem ix in that its internal intervals are equal. It should be applied also to skewed unimodal listributions which can be transformed to symmetrical form, for example by taking logarithms, osines or reciprocals. In addition to reference to the mean, a standard deviation basis has the dvantage, compared with a range basis, of an open-ended class for each tail of the frequency listribution. The extension of these classes permits greater discrimination near the mode.

Use of a standard deviation basis does not imply a class interval of *one* standard deviation, which is too coarse if more than four classes are employed. Table I shows the proportions of a normal distribution which fall into each of 5, 6 or 7 classes, for different widths of class. For a -class division, classes 0·5 to 0·6 standard-deviations wide are occupied almost equally, and the esult approaches a percentile-based solution. This excessive pooling in the broad, open-ended ail classes tends to suppress the bell-shaped nature of the frequency distribution. On the other

TABLE I

Percentage of values in a normal distribution falling into each class, for different class widths expressed as proportions of standard deviation. Class limits are shown in brackets in units of standard deviation from the mean. Based on tables of the normal distribution

Class width (standard deviations)	Class *1*		*2*		*3*		*4*		*5*		*6*		*7*
(a) *Four classes*													
1·1	13·6	(−1·1)	36·4	(0·0)	36·4	(+1·1)	13·6						
1·0	15·9	(−1·0)	34·1	(0·0)	34·1	(+1·0)	15·9						
0·9	18·4	(−0·9)	31·6	(0·0)	31·6	(+0·9)	18·4						
0·8	21·2	(−0·8)	28·8	(0·0)	28·8	(+0·8)	21·2						
0·7	24·2	(−0·7)	25·8	(0·0)	25·8	(+0·7)	24·2						
0·6	27·4	(−0·6)	22·6	(0·0)	22·6	(+0·6)	27·4						
0·5	30·9	(−0·5)	19·1	(0·0)	19·1	(+0·5)	30·9						
(b) *Five classes*													
1·1	5·0	(−1·65)	24·2	(−0·55)	41·8	(+0·55)	24·2	(+1·65)	5·0				
1·0	6·7	(−1·5)	24·1	(−0·5)	38·3	(+0·5)	24·1	(+1·5)	6·7				
0·9	8·8	(−1·35)	23·8	(−0·45)	34·7	(+0·45)	23·8	(+1·35)	8·8				
0·8	11·5	(−1·2)	22·9	(−0·4)	31·1	(+0·4)	22·9	(+1·2)	11·5				
0·7	14·7	(−1·05)	21·6	(−0·35)	27·4	(+0·35)	21·6	(+1·05)	14·7				
0·6	18·4	(−0·9)	19·8	(−0·3)	23·6	(+0·3)	19·8	(+0·9)	18·4				
0·5	22·7	(−0·75)	17·9	(−0·25)	19·7	(+0·25)	17·9	(+0·75)	22·7				
(c) *Six classes*													
1·1	1·4	(−2·2)	12·2	(−1·1)	36·4	(0·0)	36·4	(+1·1)	12·2	(+2·2)	1·4		
1·0	2·3	(−2·0)	13·6	(−1·0)	34·1	(0·0)	34·1	(+1·0)	13·6	(+2·0)	2·3		
0·9	3·6	(−1·8)	14·8	(−0·9)	31·6	(0·0)	31·6	(+0·9)	14·8	(+1·8)	3·6		
0·8	5·5	(−1·6)	15·7	(−0·8)	28·8	(0·0)	28·8	(+0·8)	15·7	(+1·6)	5·5		
0·7	8·1	(−1·4)	16·1	(−0·7)	25·8	(0·0)	25·8	(+0·7)	16·1	(+1·4)	8·1		
0·6	11·5	(−1·2)	15·9	(−0·6)	22·6	(0·0)	22·6	(+0·6)	15·9	(+1·2)	11·5		
0·5	15·9	(−1·0)	15·0	(−0·5)	19·1	(0·0)	19·1	(+0·5)	15·0	(+1·0)	15·9		
(d) *Seven classes*													
1·1	0·05	(−3·3)	1·34	(−2·2)	12·18	(−1·1)	72·85	(+1·1)	12·18	(+2·2)	1·34	(+3·3)	0·05
1·0	0·13	(−3·0)	2·15	(−2·0)	13·59	(−1·0)	68·26	(+1·0)	13·59	(+2·0)	2·15	(+3·0)	0·13
0·9	0·35	(−2·7)	3·24	(−1·8)	14·82	(−0·9)	63·18	(+0·9)	14·82	(+1·8)	3·24	(+2·7)	0·35
0·8	0·82	(−2·4)	4·66	(−1·6)	15·71	(−0·8)	57·62	(+0·8)	15·71	(+1·6)	4·66	(+2·4)	0·82
0·7	1·79	(−2·1)	6·29	(−1·4)	16·12	(−0·7)	51·60	(+0·7)	16·12	(+1·4)	6·29	(+2·1)	1·79
0·6	3·59	(−1·8)	7·92	(−1·2)	15·92	(−0·6)	45·14	(+0·6)	15·92	(+1·2)	7·92	(+1·8)	3·59
0·5	6·68	(−1·5)	9·19	(−1·0)	14·98	(−0·5)	38·30	(+0·5)	14·98	(+1·0)	9·19	(+1·5)	6·68
0·4	11·51	(−1·2)	9·68	(−0·8)	13·27	(−0·4)	31·08	(+0·4)	13·27	(+0·8)	9·68	(+1·2)	11·51

hand, classes larger than one standard deviation leave few measurements to the tail classes, which are thus under-utilized. Hence an intermediate value, say 0·7 or 0·8 standard deviations, will usually be preferable. It may eventually be possible to standardize this factor, for a given number of classes.

The standard deviation system is less suitable for large numbers of classes (say twenty) because frequencies may then be low in classes near the tails.

For a rectangular frequency distribution equal divisions of the range (system xi (F)) give excellent results, comparable to the use of percentiles but without the minor perturbations of class width. (Minor perturbations of class frequency are less objectionable.) Rectangular distributions are, however, rarely encountered, and occasions where range sub-division is the best method are correspondingly rare. For normal or skewed distributions, most of the measurements

TABLE II

Summary of properties of six systems for class intervals

	Class limits	Suitability for rectangular	Suitability for normal	Suitability for J-shaped	Robustness	Flexibility or subjectivity	Are 'round numbers' class limits possible?
(vii) Percentile	Data-specific	Good	Good	Good	Great	Moderate	No
(viii) Nested-mean	Data-specific but comparable	Good	Good	Good	High	Low (2^m classes only)	No No
(ix) Normal percentile	Parameter-dependent; comparable	Fair	Good	Overfills lower	Fair	Moderate	No
(x) Standard deviation	Standard (equal except open end classes)	Overfills middle	Excellent	Overfills lower	Fair	Great	No
(xiv) Geometric progression (base usually 2·0)	Standard	Overfills higher (N.B. base 1·0 gives equal intervals)	Overfills middle Underfills lower	Excellent	Lacking	Great	Yes (if not data-calibrated)
(xi) Equal interval	Standard	Excellent	Overfills middle	Overfills lower	Lacking	Moderate	Yes (if not data-calibrated)

Note: Flexibility is moderate if any number of classes can be defined, great if a further parameter can be varied. Class limits are 'data-specific' if dependent on individual data values: 'standard' if they form a numerical series independent of the data, even if they are calibrated in relation to statistical parameters of the data; and 'comparable' if they stand in mathematical relation to each other, even though not standard. Most class-interval systems (and especially nested means) can be envisaged as compromise between percentile classes, with the desirable property of placing equal numbers of symbols in each class, and equal-interval classes, with the desirable property of equal width.

fall into one or two of the range sub-divisions, while classes covering tails of the distribution are hardly used. The range-based technique is easily operationalized, and hence it is the 'default' option used by many computer programs, unless the user intervenes. The adverse results of such cartographic carelessness have been noted by Hsu and Porter (1971). Furthermore, the example they quote gives class limits to five significant digits! In the rare cases where system xi is appropriate, it might be better (Schultz, 1961) to use rounded numbers (system xi (a)), at the expense of incomplete use of the highest and perhaps the lowest class.

Equal-interval classes on the various scales mentioned in systems xii and xiii are rarely used, but would be appropriate to frequency distributions approximately rectangular on the scale in question. If they were unimodal and near-symmetric on the same scale, system x would make more economic use of the classes available. Standard-deviation class limits are also equally spaced, but the open-ended extreme classes are useful for the tails of a distribution.

Geometric progressions of class widths (system xiv) are extremely useful for distributions where frequency declines continuously with increasing magnitude ('J-shaped' distributions). They are also very flexible in that different-sized lowest classes and different bases of the progression can be used. Since they have not received detailed cartographic attention before, they are considered at length below. In view of the flexibility of geometric progressions, with class width changing slowly or rapidly depending on the base of the geometric progression, arithmetic progressions appear superfluous: they seem to have no particular rationale. Mackay's (1963) system xvi seems unnecessarily complex, at least until geometric progressions have been fully explored and found wanting.

Implicit in the above discussion is the principle that if serial class-interval systems are selected carefully to suit the overall shape of a frequency distribution, there is no need to forgo their advantages for the irregularity of idiographic systems. Any irregular system of class limits distorts the form of the statistical surface (Robinson and Sale, 1969, p. 166). Idiographic systems (especially iii) take quantitative data and degrade them into grouped ordinal data, emphasizing differences between groups rather than quantitative differences on a continuous scale.

Table II compares the suitability of six important class-interval systems, including two idiographic ones. It is suggested on the basis of the preceding discussion that intervals based on standard deviation are best for normal frequency distributions, equal-intervals for rectangular, and geometric progressions for J-shaped. Most distributions which do not approximate to any of these forms can be transformed to one of them. Significantly multimodal distributions are an exception: class limits must be located in the intervening breaks and if there are more than two modes this makes the use of serial intervals fraught with difficulty.

Nested means are attractive because of their robustness and because the relative frequency of each class provides the map reader with clues to the shape of frequency distribution. There is, however, no particular advantage in using a single system when the limits are irregular and non-serial; the combination proposed above seems preferable to nested means alone. Percentiles have been selected by cartographers wishing to play safe and make sure that some spatial differentiation was portrayed. With the use of computers there is no excuse for ignoring the frequency distribution and Hunt's (1968, p. 3) practical reasons for using percentiles, rather than the standard deviation basis which he would have preferred, no longer apply.

CLASS INTERVALS AND MAP COMPARISON

A number of class-interval systems can be claimed to provide comparison between maps of the same variable for different areas or times, or between maps of different variables for the same area. Percentile classes permit us to compare equal divisions of the frequency distribution. Nested means provide comparably defined benchmarks in each distribution. Standard-deviation rmit us to compare dispersion-standardized deviations from respective means. 'Natural ss limits would fall in comparable positions, in the unlikely event of their having any nificance. Arbitrary round-number class limits can be repeated for different maps of any n the same units, although some classes may be vacant on some maps.

vital question is why are the maps being compared. It may be impossible to optimize fication on each map and at the same time facilitate all possible modes of map compari- aps of different times are to be compared to reveal changes in absolute terms, or if maps nt variables in the same units are to be compared for differences in absolute terms, then ntial that exactly the same class limits should be used, and each individual class must be ed identically on all maps. In this case, none of the data-calibrated systems can be used and there are two alternatives: (a) use round-number class limits equally spaced on an ate scale or perhaps a geometric progression; or (b) apply a data-calibrated system to the data of the two or more periods or variables. In either case one must tolerate the fact vidual maps are in no way optimized for the data portrayed. Optimum use is not made of ber of classes available, and, in fact, the number of classes may vary from map to map: eferable to varying the class limits.

vever, there are further motives for map comparison: a major one is, for a given area, to far the spatial pattern of one variable has changed over time, or how similar are the atterns of different variables. Here the use of identical class limits would actually be a e, because these would interact with differences in mean and in standard deviation produce apparent differences even when the spatial patterns were identical. Hence, such son requires that class intervals should be related to the data in a standard way, and umbers of classes should be used. The above reasons justifying the use of a percentile, nean, standard-deviation, or natural-break basis are applicable. Equal division of the l range would be inefficient because the range is an unstable statistic, and inappropriate it is unrelated to the three principal measures of central tendency.

ether variables with different-shaped frequency distributions are best compared through les, nested means or appropriate data-calibrated serial systems is a debatable question; entation would be useful. Although Olson (1971, 1972a) suggested that percentile classes st in preserving the rank correlation between pairs of maps (compared with correlation the data sets), her only reasonable comparison was for four-class maps. The results n her (1972a) figure 3 indicate that sampling variation of rank correlation is least with neans, slightly worse for standard deviations (limits at −1, 0 and +1 S.D.), and consider- rse for quantiles. Three-class and five-class standard-deviation systems *appeared* worse rresponding percentile divisions because Olson selected very inefficient class limits: 1, +1 and +2 standard deviations. The central class, where observations are most spaced, was thus twice as wide as those adjacent to it, and contained some 68 per cent of ervations: it should never be wider than the adjacent classes, for unimodal frequency tions.

son (1972b) then compared similar class-interval systems for 900 pairs of real demographic s which were presumed to cover a range of deviations from normality in frequency distri- Errors were greater than for simulated normal data, and percentile techniques were that even two standard-deviation classes were better than (five) quintiles! For equal s of classes, standard-deviation systems (even as defined by Olson) were much better, means were intermediate. The use of deviations from expected rank correlation, rather viations from average rank correlation, reduced the apparent merit of nested means for ed data also; this suggests that their portrayed correlations are biased as well as imprecise. omparisons might be obtained by using systems from Table I with more nearly equal

class frequencies: Olson's existing results argue against percentiles, rather than for them. As yet, considerations of map comparison provide no basis for modification of the recommendations made at the end of the last section.

THE ACCURACY OF GRADED SYMBOLIZATION

Measures of the classification accuracy of maps have been proposed as means of selecting the best class-interval system. Jenks and Caspall (1971) have made a serious attempt to measure the accuracy of graded choropleth maps, following the pioneer work of Jenks and Coulson (1963). The latter measured error as

$$D = \sum_{j=1}^{N} \left| \frac{R_j}{\bar{Z}_j} - \frac{R_j}{M_j} \right| \tag{1}$$

where R_j is the theoretical (complete) range of class j, M_j is its midpoint, and $\bar{Z}_j$ is the mean of the observations which fall into it. Thus the greater the difference between the reciprocals of M_j and $\bar{Z}_j$, the greater the class error: the summation of these errors is weighted by the class range. Broad classes thus dominate the calculation, for no obvious reason. Though the results do have some relation to map accuracy, they are peculiar in ignoring the dispersion of observations (around $\bar{Z}_j$) within each class. Unfortunately, D is the measure used by Chang (1974) in his class-interval selection program; it should now be apparent that it is not a suitable measure of accuracy.

Jenks and Caspall (1971) distinguished accuracy in three map functions; tabular (place-by-place), overview (volumetric) and boundary.

The tabular accuracy index (TAI) seems an interesting and readily operated measure. It is based on Jenks's (1963) concept that a choropleth map implicitly represents each place in a given class by the average value for all places in that class.

$$\mathrm{TAI} = 1 - \frac{\sum_{j=1}^{N} \sum_{k=1}^{n_j} \left| Z_{kj} - \bar{Z}_j \right|}{\sum_{j=1}^{N} \sum_{k=1}^{n_j} \left| Z_{kj} - \bar{Z} \right|} \tag{2}$$

where $\bar{Z}$ is the mean of all n observations, $\bar{Z}_j$ is the mean of the n_j observations in class j, Z_{kj} is an individual observation in class j, and N is the number of classes.

Unfortunately, as Jenks and Caspall pointed out (1971, p. 220), the map reader is not usually given $\bar{Z}_j$. Despite their own recommendation, Jenks and Caspall did not even give $\bar{Z}_j$ for the numerous maps in the same article. The mean of observations in a class is therefore irrelevant to the value which a map reader is likely to put on a class. The reader, since he is usually given the class limits, is presumably most likely to visualize the class mid-point, M_j, though even this involves him in a small mental calculation. Alternatively, he might envisage the places mapped in a particular class as being spread evenly through the class, though he would not know in what order they came. Use of M_j in place of $\bar{Z}_j$ in the TAI would reduce the apparent accuracy of classes where the observations clustered together but not around the class midpoint. This presumably has implications for the apparent merits of different class-interval systems. All such indices have been based on the original measurement scale; if it is appropriate to give less weight to large differences in the tail of a skewed distribution, the index should be applied only *after* a transformation toward symmetry.

The overview accuracy index is simply an area-weighted version of the tabular; it relates to the volumetric mismatch between the true three-dimensional model and the apparent one. This does not seem to be a full solution to its authors' apparent aim of measuring success in ing a synoptic view of the pattern, form or structure of the spatial distribution: indeed, ns unlikely that such a measure can be derived outside the context of map perception .

he third index asks the reasonable question, to what extent do true contrasts between ours remain apparent in a choropleth map. Unlike the TAI, the boundary accuracy index would be difficult to apply to point symbols, and impossible for isopleth maps. Like the the BAI is based on mean values of observations per class, and the apparent difference en neighbours is taken as the difference between these means for the two classes into they fall, despite the fact that the map reader is unaware of such means. The sum of the differences is divided by the sum of the p largest true differences, giving the BAI. Again, uld be improved by replacing Z by M_j in calculating apparent differences. Also, the BAI s error due to very different neighbouring values being placed in the same class. So that ons with different numbers of classes, and even different numbers of contrasting con-es, could be compared more readily, it would be better to divide by the sum of *all* true ences between neighbours, not just the p largest. This would give:

$$\frac{\Sigma^* \left| M_{xj} - M_{yj} \right|}{\Sigma^* \left| \quad z_x - z_y \quad \right|} \tag{3}$$

M_{xj} is the class midpoint appropriate to x, M_{yj} that appropriate to y, and the summations over values of x and y which give contiguous pairs of symbols or spatial divisions.

ven after such improvements, however, the BAI is misleading, because cases where the ent contrast exceeds the true contrast increase the accuracy index; surely they should ssively decrease it. Hence we arrive at a rather different 'neighbour difference error (NDEI) which compares the apparent contrast with the true contrast, and standardizes m of differences (positive or negative) by the sum of true contrasts;

$$\mathrm{NDEI} = \frac{\Sigma^* \left| (M_{xj} - M_{yj}) - (Z_x - Z_y) \right|}{\Sigma^* \left| Z_x - Z_y \right|} \tag{4}$$

uch a boundary index is quite useful in assessing the faithfulness of the map in portraying l trends, as well as more localized contrasts. The NDEI cannot be negative: it varies from or a perfect map (which would need an infinite number of classes) to one for a one-class where the $(M_{xj} - M_{yj})$ term vanishes.

lternatively, a cross-product model could be employed in place of the difference model, a 'neighbour cross-product error index':

$$\mathrm{NCEI} = \frac{\Sigma^* (M_{xj} - M_{yj})(Z_x - Z_y)}{\Sigma^* (Z_x - Z_y)} \tag{5}$$

er improvements can be made by using root-mean-square measures in place of mean-ion measures. If such accuracy indices are carefully defined, they are essential components idiographic approach. They are almost irrelevant to the choice of serial or exogenous als, except as checks on the degree of 'suboptimality' which has been accepted. They might elp decide on the number of classes to be shown.

final recommendation is that a combination of different accuracy indices, as used in Jenks Caspall's (1971) map accuracy index, is of no general value. Rather, each map designer decide what weight to give to each index—if he feels that idiographic optimization of this is appropriate.

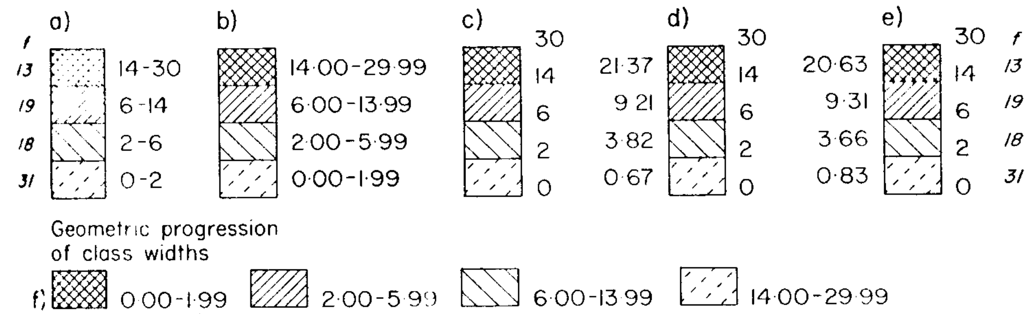


FIGURE 1 Six alternative legends representing a geometric progression of class widths, each class being twice as wide as the next lower class

PRESENTATION OF LEGENDS TO CLASSES

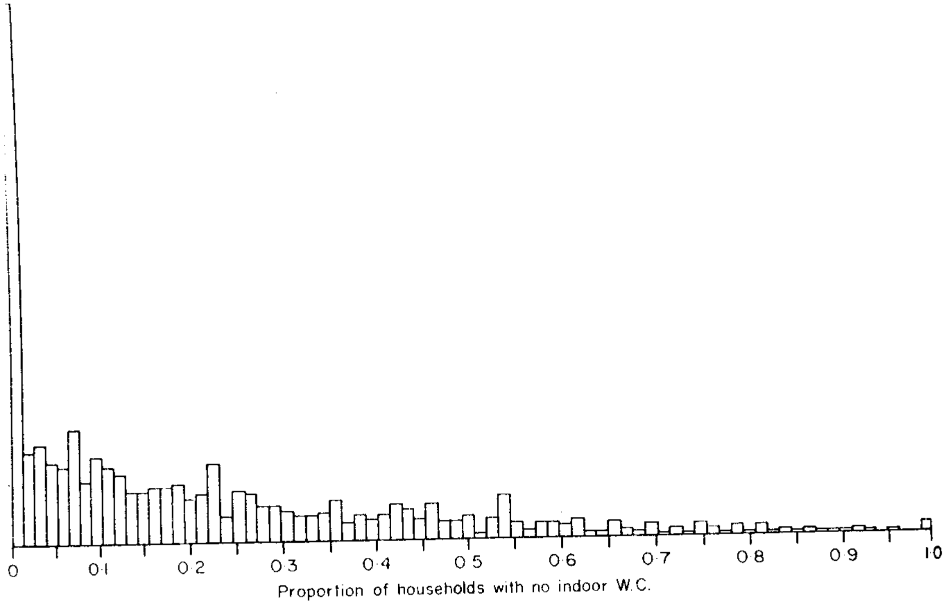
To select a good class-interval system is not enough; the intervals must be communicated to the reader. Since a map should be able to convey its message rapidly, the legend should be almost redundant, serving to provide at a glance (if the information is quantitative) a calibration of the absolute levels involved. (The relative levels should be obvious from the symbolism selected.) Hence the class limits should be indicated as simply as possible, and so that their progression is readily perceived.

Figure 1 legend (a) is clumsy in repeating each internal class limit, thus increasing visual clutter, and not indicating whether values of exactly 2, 6 and 14 are assigned to the class above or to that below. For data with a resolution of two decimal places, legend (b) meets the latter problem, but not the former: in fact it takes considerably longer to read (Schultz, 1961, p. 228). The preferred form is legend (c), which indicates the limits at the bottom of the classes to which they are assigned. A reader has to scan vertically to find the two limits defining a class interval, but this is desirable, since it leads him on to consider the complete series and to envisage its serial nature and the continuity of the quantitative scale. An addition which could be made to any of these legends is the indication of number of observations (f) per class on the left-hand side: unlike the class limits on the right-hand side, these numbers are centred on each class box. The name of the variable is best placed to the right of the class limits. Below the legend the nature of the class-interval system may be indicated: if the map is for a specialist audience, the fact that the progression is 'to the base 2' can be added.

Version (d) shows the average of the observed values in each class, while (e) shows the class midpoints, which in this case are the extra boundaries needed for an eight-class division of the same 0–30 range, with base $2^{0.5}$. The latter might be the more useful way of indicating central values for each class. If class frequencies are also to be shown, they should be located to the right of the class limits as in (e) and printed in a thinner, distinct typeface so that they do not obscure the vertical progression of class limits. Finally, legend (f) is a variant which obscures the continuity of the scale and seems inefficient for a quantitative map, but might be appropriate for a map of nominal-scale data.

A further improvement would be to give a histogram as another insert, and to mark the class limits thereon. If nested means are used, the hierarchic nature of class limits should be apparent on the legend. If multiples of the standard deviation are used, the mean and standard deviation should be given next to the legend. On the whole, (c) appears the clearest style of legend.

The presentation of legends to proportionally symbolized maps is more problematic. For proportional point symbols such as circles or squares, the use of a continuous line spanning the



E 2 Histogram of proportion of households with no indoor W.C., for 1043 1 × 1 km grid squares with eight or more households in County Durham, 1971: a mildly J-shaped frequency distribution

rs of successive legend symbols of increasing size does suggest proportionality rather ading, as does presentation of the equation by which proportionality is defined. The legend values should always be close to the extreme map values. Dobson's (1974) ion of idiographic grouping, on the other hand, is likely to mislead the reader into proportional symbols as graded ones. The necessary number of circles in the legend e reduced by one or two, at the expense of confusing the reader with an irregular series smooth progression would serve much better.

r choropleth maps, the equivalent to the continuous spanning line is perhaps the con- shading box with convergent lines used for a legend by Tobler (1973). Two difficulties is are that the angular relations between shading lines are falsified, and the map reader has e area of a given shading density that might be compared with such areas on the face of the here shading density changes only at the boundaries of spatial divisions. If dot shading were a continuous-shading box legend, the first difficulty could be overcome by placing dots ly but with a continuously increasing probability towards the top end of the scale. The difficulty could be met only by having discrete shading boxes as samples at a series of in the scale. These would differ from those for graded shading in having single magnitude placed next to the middle of each box, not at the lower or upper margins. Again, propor- y should be emphasized by giving the equation relating shading densities to data values.

GEOMETRIC PROGRESSIONS

vkward type of frequency distribution is 'J-shaped', with a mode at one end of the scale y at zero) and a long tail (Fig. 2). This is extremely difficult to normalize by a monotonic rmation, which can only be effective if there is a decline in frequency on both sides of the

mode. The usual solution is a geometric progression of class limits, that is the use of equal intervals on a logarithmic scale. J-shaped distributions often remain J-shaped on a logarithmic scale, so that the lower classes may contain more measurements than the highest. Having rather fewer values in the higher classes gives a suggestion of the J-shaped nature of the distribution, and caters also for Mackay's (1955) point that large areas of the densest shading are aesthetically displeasing. If, on the other hand, the J-shaped distribution is negatively skewed, that is piled up against an upper limit such as 100 per cent, geometric intervals are selected by measuring downward from the upper limit.

Many published maps are based on geometric progressions of class upper limits, but these do not in general provide geometric progressions of class *intervals* (widths). The latter is essential since each successive class must be broader to cope with the decreasing frequency found in a J-shaped distribution. For example, limits at 2, 4, 8, 16 and 32 provide a geometric progression *only* if 1 is the lowest possible measurement. If the lowest measurement is 0, both the lower classes (0–2 and 2–4) are of equal width, so there is a break in the progression of class intervals and the lowest class is likely to contain many more measurements than any other. In fact, on a logarithmic scale the width of the 0–2 class is infinite. If the class *intervals* are to have the geometric progression 2, 4, 8, 16, 32, the limits must then fall at 0, 2, 6, 14, and 30, i.e. 2 units below the previous values (2 units being the size of the smallest class). Geometric progressions of class width do *not* give equal divisions of a logarithmic scale. In addition to taking a different-sized smallest class, the progression can be changed from base 2 (doubling) to base 3 (tripling, which gives class widths of 2, 6, 18, 54 and 162) or to a fractional base such as 3·4.

If a is the size of the first class, and x is the base of the geometric progression, successive class widths for N classes are: $a, ax, ax^2, ax^3, ax^4, \ldots, ax^{N-1}$.

If the lower limit of the first class is zero, the upper limit of class j is (Armand, 1973, p. 498):

$$\sum_{c=0}^{c=j-1} ax^c = a \sum_{c=0}^{c=j-1} x^c = \frac{a(1-x^j)}{(1-x)} \tag{6}$$

Table III lists a number of possible geometric progressions. It applies only to measurements on continuous scales, such as ratios. For discrete measurements, such as counts, class width must be expressed as the number of different measurements which can fall into the class. Hence, if measurement was orginally to 0·5°, an initial class width of one measurement combined with a progression to the base 2 gives the following classes: 0·0°; 0·5 and 1·0°; 1·5, 2·0, 2·5 and 3·0°; 3·5 to 7·0°; 7·5 to 15·0°; and so on, doubling the number of 0·5° units in each successive class. Strictly, the class limits fall at the intermediate values 0·25°, 1·25°, 3·25°, 7·25° etc., but it is easier to think of the highest possible measurement in each class: 0·0°, 1·0°, 3·0°, 7·0°, 15·0°, etc.

A troubling aspect of the use of such geometric progressions is that even when the number of classes is pre-ordained, two further decisions have to be made on a subjective basis; the size of the first class, and the base of the progression. This is usually done by trial and error, to ensure that a reasonable number of observations fall in each class. In fact, some distributions are so J-shaped that it is convenient, if inelegant, to have an open-ended sparsely occupied upper class which includes a few observations which would fall into an extra, uppermost class if the progression were continued.

One solution is to take the logarithmic range and divide it into as many equal parts as there are classes (Jenks and Coulson, 1963). This is undesirable, first, because the range depends on the maximum, which is especially capricious in a J-shaped distribution; and, second, because it cannot be used if any zero values occur, since these make the logarithmic range infinite. The latter prevents its use for most occurrences of J-shaped distributions, for example number or

TABLE III

Class limits for geometric progressions of class width for bases from 0·1 to 4·0, up to ten classes, starting at zero with a lowest class from 0 to 1

Base	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0·1	1·00	1·10	1·11	1·11	1·11	1·11	1·11	1·11	1·11	1·11
0·2	1·00	1·20	1·24	1·25	1·25	1·25	1·25	1·25	1·25	1·25
0·3	1·00	1·30	1·39	1·42	1·43	1·43	1·43	1·43	1·43	1·43
0·4	1·00	1·40	1·56	1·62	1·65	1·66	1·66	1·67	1·67	1·67
0·5	1·00	1·50	1·75	1·87	1·94	1·97	1·98	1·99	2·00	2·00
0·6	1·00	1·60	1·96	2·18	2·31	2·38	2·43	2·46	2·47	2·48
0·7	1·00	1·70	2·19	2·53	2·77	2·94	3·06	3·14	3·20	3·24
0·8	1·00	1·80	2·44	2·95	3·36	3·69	3·95	4·16	4·33	4·46
0·9	1·00	1·90	2·71	3·44	4·10	4·69	5·22	5·70	6·13	6·51
1·0	1·00	2·00	3·00	4·00	5·00	6·00	7·00	8·00	9·00	10·00
1·1	1·00	2·10	3·31	4·64	6·11	7·72	9·49	11·44	13·58	15·94
1·2	1·00	2·20	3·64	5·37	7·44	9·93	12·02	16·50	20·80	25·96
1·3	1·00	2·30	3·99	6·19	9·04	12·76	17·58	23·86	32·01	42·62
1·4	1·00	2·40	4·36	7·10	10·95	16·32	23·85	34·39	49·15	69·81
1·5	1·00	2·50	4·75	8·12	13·19	20·78	32·17	49·26	74·89	113·33
1·6	1·00	2·60	5·16	9·26	15·81	26·30	43·07	69·92	112·87	181·59
1·7	1·00	2·70	5·59	10·50	18·86	33·05	57·19	98·23	167·98	286·57
1·8	1·00	2·80	6·04	11·87	22·37	41·27	75·28	136·50	246·70	445·06
1·9	1·00	2·90	6·51	13·37	26·40	51·16	98·21	187·60	357·43	680·12
2·0	1·00	3·00	7·00	15·00	31·00	63·00	127·00	255·00	511·00	1023·00
2·1	1·00	3·10	7·51	16·77	36·22	77·06	162·83	342·94	721·16	1515·44
2·2	1·00	3·20	8·04	18·69	42·11	93·65	207·03	456·47	1005·22	2212·49
2·3	1·00	3·30	8·59	20·76	48·74	113·10	261·14	601·62	1384·73	3185·89
2·4	1·00	3·40	9·16	22·98	56·16	135·79	326·89	785·54	1886·29	4528·10
2·5	1·00	3·50	9·75	25·38	64·44	162·09	406·23	1016·59	2542·46	6357·16
2·6	1·00	3·60	10·36	27·94	73·63	192·45	501·36	1304·54	3392·81	8822·32
2·7	1·00	3·70	10·99	30·67	83·82	227·31	614·73	1660·76	4485·06	12110·65
2·8	1·00	3·80	11·64	33·59	95·06	267·16	749·05	2098·34	5876·36	16454·82
2·9	1·00	3·90	12·31	36·70	107·43	312·54	907·36	2632·35	7634·81	22141·96
3·0	1·00	4·00	13·00	40·00	121·00	364·00	1093·00	3280·00	9841·00	29524·00
3·1	1·00	4·10	13·71	43·50	135·85	422·14	1309·65	4060·91	12589·82	39029·44
3·2	1·00	4·20	14·44	47·21	152·07	487·61	1561·35	4997·33	15992·44	51176·81
3·3	1·00	4·30	15·19	51·13	169·72	561·07	1852·54	6114·39	20178·47	66589·96
3·4	1·00	4·40	15·96	55·26	188·90	643·25	2188·06	7440·39	25298·33	86015·32
3·5	1·00	4·50	16·75	59·63	209·69	734·91	2573·17	9007·10	31525·86	110341·49
3·6	1·00	4·60	17·56	64·22	232·18	836·84	3013·62	10850·04	39061·14	140621·09
3·7	1·00	4·70	18·39	69·04	256·46	949·90	3515·63	13008·81	48133·61	178095·35
3·8	1·00	4·80	19·24	74·11	282·63	1074·98	4085·91	15527·47	59005·39	224221·49
3·9	1·00	4·90	20·11	79·43	310·77	1213·02	4731·76	18454·86	71974·95	280703·31
4·0	1·00	5·00	21·00	85·00	341·00	1365·00	5461·00	21845·00	87381·00	349525·00

TABLE IV

The progression of class widths

(i)	a		ax		ax^2	ax^3		ax^4	
(ii)	b	$bx^{1/2}$	$bx^{2/2}$	$bx^{3/2}$	$bx^{4/2}$ P $bx^{5/2}$	$bx^{6/2}$	$bx^{7/2}$	$bx^{8/2}$	$bx^{9/2}$

TABLE V

Class limits for geometric progressions of class width for five classes with a median of 10, starting at zero

Base	*1*	*2*	*3*	*4*	*5*
0·1	9·03	9·93	10·02	10·03	10·03
0·2	8·15	9·77	10·10	10·17	10·18
0·3	7·36	9·57	10·23	10·43	10·49
0·4	6·68	9·35	10·41	10·84	11·01
0·5	6·07	9·11	10·63	11·39	11·77
0·6	5·55	8·87	10·87	12·07	12·79
0·7	5·08	8·64	11·13	12·88	14·10
0·8	4·68	8·42	11·41	13·81	15·72
0·9	4·32	8·20	11·70	14·85	17·68
1·0	4·00	8·00	12·00	16·00	20·00
1·1	3·72	7·80	12·30	17·25	22·69
1·2	3·46	7·62	12·61	18·59	25·77
1·3	3·24	7·44	12·91	20·02	29·27
1·4	3·03	7·28	13·22	21·54	33·19
1·5	2·85	7·12	13·53	23·14	37·56
1·6	2·68	6·97	13·83	24·81	42·38
1·7	2·53	6·83	14·14	26·56	47·68
1·8	2·39	6·69	14·44	28·38	53·47
1·9	2·26	6·56	14·74	30·26	59·76
2·0	2·15	6·44	15·03	32·21	66·57
2·1	2·04	6·33	15·32	34·22	73·91
2·2	1·94	6·21	15·61	36·29	81·79
2·3	1·85	6·11	15·90	38·42	90·23
2·4	1·77	6·01	16·19	40·61	99·23
2·5	1·69	5·91	16·47	42·85	108·82
2·6	1·62	5·82	16·74	45·15	119·00
2·7	1·55	5·73	17·02	47·50	129·79
2·8	1·49	5·64	17·29	49·89	141·19
2·9	1·43	5·56	17·56	52·34	153·22
3·0	1·37	5·48	17·82	54·84	165·88
3·1	1·32	5·41	18·08	57·38	179·20
3·2	1·27	5·34	18·34	59·97	193·18
3·3	1·22	5·27	18·60	62·61	207·83
3·4	1·18	5·20	18·85	65·29	223·16
3·5	1·14	5·13	19·11	68·01	239·18
3·6	1·10	5·07	19·35	70·78	255·90
3·7	1·07	5·01	19·60	73·59	273·33
3·8	1·03	4·95	19·84	76·44	291·49
3·9	1·00	4·89	20·08	79·33	310·37
4·0	0·97	4·84	20·32	82·26	330·00

proportion of immigrants or other minority groups, per area. Finally, this solution does not give a geometric progression of class widths, but rather of class limits.

Instead, it is suggested that geometric progressions should pivot about some measure of central tendency. Given the high skewness, the only such obvious measure is the median. The median should be made the geometric midpoint of the middle class if there are an odd number of classes, or the boundary between the two central classes if there are an even number.

This balancing point may be established in more general terms by considering the geometric progression to the base $x^{\frac{1}{2}}$ which has twice as many classes and utilizes the same class boundaries as for base x, *plus* the geometric midpoint of each class (Table IV).

TABLE VI

Class limits for geometric progressions of class width for six classes with a median of 10, starting at zero

Base	*1*	*2*	*3*	*4*	*5*	*6*
0·1	9·01	9·91	10·00	10·01	10·01	10·01
0·2	8·06	9·68	10·00	10·06	10·08	10·08
0·3	7·19	9·35	10·00	10·19	10·25	10·27
0·4	6·41	8·97	10·00	10·41	10·57	10·64
0·5	5·71	8·57	10·00	10·71	11·07	11·25
0·6	5·10	8·16	10·00	11·10	11·76	12·16
0·7	4·57	7·76	10·00	11·57	12·66	13·43
0·8	4·10	7·38	10·00	12·10	13·78	15·12
0·9	3·69	7·01	10·00	12·69	15·11	17·29
1·0	3·33	6·67	10·00	13·33	16·67	20·00
1·1	3·02	6·34	10·00	14·02	18·44	23·31
1·2	2·75	6·04	10·00	14·75	20·44	27·28
1·3	2·51	5·76	10·00	15·51	22·66	31·97
1·4	2·29	5·50	10·00	16·29	25·10	37·44
1·5	2·11	5·26	10·00	17·11	27·76	43·75
1·6	1·94	5·04	10·00	17·94	30·64	50·96
1·7	1·79	4·83	10·00	18·79	33·73	59·13
1·8	1·66	4·64	10·00	19·66	37·04	68·32
1·9	1·54	4·45	10·00	20·54	40·55	78·59
2·0	1·43	4·29	10·00	21·43	44·29	90·00
2·1	1·33	4·13	10·00	22·33	48·23	102·61
2·2	1·24	3·98	10·00	23·24	52·38	116·48
2·3	1·16	3·84	10·00	24·16	56·74	131·67
2·4	1·09	3·71	10·00	25·09	61·31	148·24
2·5	1·03	3·59	10·00	26·03	66·09	166·25
2·6	0·97	3·47	10·00	26·97	71·07	185·76
2·7	0·91	3·37	10·00	27·91	76·27	206·83
2·8	0·86	3·26	10·00	28·86	81·66	229·52
2·9	0·81	3·17	10·00	29·81	87·27	253·89
3·0	0·77	3·08	10·00	30·77	93·08	280·00
3·1	0·73	2·99	10·00	31·73	99·09	307·91
3·2	0·69	2·91	10·00	32·69	105·31	337·68
3·3	0·66	2·83	10·00	33·66	111·73	369·37
3·4	0·63	2·76	10·00	34·63	118·36	403·04
3·5	0·60	2·69	10·00	35·60	125·19	438·75
3·6	0·57	2·62	10·00	36·57	132·22	476·56
3·7	0·54	2·56	10·00	37·54	139·46	516·53
3·8	0·52	2·49	10·00	38·52	146·89	558·72
3·9	0·50	2·44	10·00	39·50	154·54	603·19
4·0	0·48	2·38	10·00	40·48	162·38	650·00

Two progressions are shown at approximately logarithmic scale:

(i) five classes with initial class width a, progressing to the base x

(ii) ten corresponding classes with initial class width b, progressing to the base $x^{\frac{1}{2}}$

P is the median or balancing point of the progression.

The range embraced by the progression is given by

$$\sum_{c=0}^{c=4} a x^c = \sum_{c=0}^{c=9} b\, x^{c/2} = b(1-x^5)/(1-x^{\frac{1}{2}}) \qquad (7)$$

From Table IV, it can be seen that if alternate class boundaries are to coincide, the relationship

$$a = b(1+x^{\frac{1}{2}}) \qquad (8)$$

TABLE VII

Class limits for geometric progressions of class width for seven classes with a median of 10, starting at zero

Base	*1*	*2*	*3*	*4*	*5*	*6*	*7*
0·1	9·00	9·90	9·99	10·00	10·00	10·00	10·00
0·2	8·03	9·63	9·96	10·02	10·03	10·04	10·04
0·3	7·11	9·24	9·88	10·07	10·13	10·14	10·15
0·4	6·25	8·75	9·75	10·16	10·32	10·38	10·40
0·5	5·48	8·23	9·60	10·28	10·63	10·80	10·88
0·6	4·80	7·69	9·42	10·45	11·08	11·45	11·67
0·7	4·21	7·15	9·21	10·66	11·67	12·37	12·87
0·8	3·69	6·64	9·00	10·89	12·40	13·61	14·58
0·9	3·24	6·16	8·79	11·15	13·28	15·19	16·92
1·0	2·86	5·71	8·57	11·43	14·29	17·14	20·00
1·1	2·53	5·30	8·36	11·72	15·42	19·49	23·96
1·2	2·24	4·93	8·15	12·02	16·67	22·24	28·93
1·3	1·99	4·58	7·95	12·33	18·03	25·43	35·05
1·4	1·78	4·27	7·76	12·65	19·49	29·06	42·47
1·5	1·60	3·99	7·58	12·96	21·04	33·16	51·33
1·6	1·44	3·73	7·40	13·28	22·69	37·73	61·81
1·7	1·29	3·50	7·24	13·60	24·42	42·80	74·06
1·8	1·17	3·28	7·08	13·92	26·22	48·37	88·24
1·9	1·06	3·09	6·93	14·23	28·10	54·46	104·54
2·0	0·97	2·91	6·79	14·54	30·06	61·08	123·14
2·1	0·89	2·75	6·65	14·85	32·08	68·25	144·20
2·2	0·81	2·60	6·52	15·16	34·16	75·97	167·94
2·3	0·74	2·46	6·40	15·46	36·31	84·25	194·52
2·4	0·69	2·33	6·28	15·76	38·51	93·11	224·16
2·5	0·63	2·21	6·17	16·06	40·77	102·57	257·05
2·6	0·59	2·11	6·06	16·35	43·09	112·62	293·40
2·7	0·54	2·01	5·96	16·64	45·46	123·29	333·42
2·8	0·50	1·91	5·86	16·92	47·88	134·58	377·33
2·9	0·47	1·83	5·77	17·20	50·36	146·50	425·33
3·0	0·44	1·75	5·68	17·48	52·88	159·07	477·65
3·1	0·41	1·67	5·60	17·75	55·45	172·30	534·52
3·2	0·38	1·60	5·51	18·03	58·06	186·18	596·17
3·3	0·36	1·54	5·43	18·29	60·72	200·75	662·83
3·4	0·34	1·48	5·36	18·56	63·43	216·00	734·73
3·5	0·32	1·42	5·29	18·82	66·18	231·94	812·12
3·6	0·30	1·37	5·22	19·08	68·97	248·59	895·23
3·7	0·28	1·32	5·15	19·33	71·81	265·96	984·33
3·8	0·26	1·27	5·08	19·58	74·68	284·05	1079·65
3·9	0·25	1·22	5·02	19·83	77·60	302·87	1181·45
4·0	0·24	1·18	4·96	20·08	80·55	322·44	1290·00

must hold. The appropriate 'balancing point' for the five-class series is the boundary, P, between the fifth and sixth classes of the ten-class series: in general terms,

$$P = b \sum_{c=0}^{c=N-1} x^{c/2} = \frac{a}{1+x^{\frac{1}{2}}} \sum_{c=0}^{c=N-1} x^{c/2} = \frac{a(1-x^{N/2})}{(1-x)} \qquad (9)$$

where N is the number of classes in the series of which P is the balancing point. By setting P equal to the median of the observed frequency distribution, we can calculate the size of the initial class:

$$a = P(1+x^{\frac{1}{2}}) \Bigg/ \left(\sum_{c=0}^{c=N-1} x^{c/2} \right) = \frac{P(1-x)}{1-x^{N/2}} \qquad (10)$$

TABLE VIII

Class limits for geometric progressions of class width for eight classes with a median of 10, starting at zero

Base	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
0·1	9·00	9·90	9·99	10·00	10·00	10·00	10·00	10·00
0·2	8·01	9·62	9·94	10·00	10·01	10·02	10·02	10·02
0·3	7·06	9·17	9·81	10·00	10·06	10·07	10·08	10·08
0·4	6·16	8·62	9·61	10·00	10·16	10·22	10·25	10·26
0·5	5·33	8·00	9·33	10·00	10·33	10·50	10·58	10·63
0·6	4·60	7·35	9·01	10·00	10·60	10·95	11·17	11·30
0·7	3·95	6·71	8·65	10·00	10·95	11·61	12·08	12·40
0·8	3·39	6·10	8·27	10·00	11·39	12·50	13·39	14·10
0·9	2·91	5·52	7·88	10·00	11·91	13·62	15·17	16·56
1·0	2·50	5·00	7·50	10·00	12·50	15·00	17·50	20·00
1·1	2·15	4·52	7·13	10·00	13·15	16·62	20·44	24·64
1·2	1·86	4·10	6·78	10·00	13·86	18·50	24·06	30·74
1·3	1·62	3·72	6·45	10·00	14·62	20·62	28·42	38·56
1·4	1·41	3·38	6·14	10·00	15·41	22·98	33·58	48·42
1·5	1·23	3·08	5·85	10·00	16·23	25·58	39·60	60·62
1·6	1·08	2·81	5·57	10·00	17·08	28·41	46·53	75·54
1·7	0·95	2·57	5·32	10·00	17·95	31·47	54·45	93·52
1·8	0·84	2·36	5·09	10·00	18·84	34·76	63·41	114·98
1·9	0·75	2·17	4·87	10·00	19·75	38·27	73·46	140·32
2·0	0·67	2·00	4·67	10·00	20·67	42·00	84·67	170·00
2·1	0·60	1·85	4·48	10·00	21·60	45·95	97·09	204·48
2·2	0·54	1·71	4·30	10·00	22·54	50·11	110·78	244·26
2·3	0·48	1·59	4·14	10·00	23·48	54·49	125·81	289·84
2·4	0·44	1·48	3·99	10·00	24·44	59·08	142·22	341·77
2·5	0·39	1·38	3·84	10·00	25·39	63·88	160·09	400·62
2·6	0·36	1·29	3·71	10·00	26·36	68·89	179·47	466·97
2·7	0·33	1·21	3·58	10·00	27·33	74·11	200·41	541·44
2·8	0·30	1·13	3·47	10·00	28·30	79·53	222·98	624·65
2·9	0·27	1·06	3·35	10·00	29·27	85·16	247·24	717·28
3·0	0·25	1·00	3·25	10·00	30·25	91·00	273·25	820·00
3·1	0·23	0·94	3·15	10·00	31·23	97·04	301·06	933·52
3·2	0·21	0·89	3·06	10·00	32·21	103·29	330·74	1058·58
3·3	0·20	0·84	2·97	10·00	33·20	109·74	362·34	1195·92
3·4	0·18	0·80	2·89	10·00	34·18	116·40	395·93	1346·33
3·5	0·17	0·75	2·81	10·00	35·17	123·25	431·56	1510·62
3·6	0·16	0·72	2·73	10·00	36·16	130·32	469·29	1689·61
3·7	0·14	0·68	2·66	10·00	37·14	137·58	509·19	1884·16
3·8	0·13	0·65	2·60	10·00	38·13	145·05	551·32	2095·13
3·9	0·13	0·62	2·53	10·00	39·13	152·72	595·72	2323·44
4·0	0·12	0·59	2·47	10·00	40·12	160·59	642·47	2569·99

Hence instead of selecting two arbitrary parameters, we need only select one, the base x, in addition to the number of classes. Tables V to VIII give such geometric progressions for five, six, seven, and eight classes respectively, for a fixed median of 10 and for series starting at 0. Any base between 0·1 and 4·0 can be selected, and the series can be applied to any data set by multiplying the tabulated values by (median /10).

For example, if a prior decision has been taken to map six classes, and a data set has a median of 4 and a maximum of 110, a series from Table V is selected and multiplied by 0·4. Base 3·0 thus gives the limits 0·0, 0·308, 1·232, 4·000, 12·308, 37·232, 112·000, which just accommodates the range of observed values. A higher base could be employed, but the highest class would then be under-used; and if a much lower base were used, the maximum would fall beyond the strict limit of the highest class, which is inelegant though sometimes practical in permitting greater differentiation among low values. Those who wish to use the maximum, despite its capricious nature, can set this equal to the upper limit of the highest class (after multiplication by (median /10)) and use the appropriate base established by interpolation. Equation (6) permits recalculation of class boundaries to any desired level of resolution. It is recommended, however, that for presentation on a map legend, class limits should be rounded to the resolution of the data, or to three or four significant digits for ratio data.

CLOSED PERCENTAGES

A further consideration is that geometric progressions are often applied to percentages, for example of minority groups. In this case there is an upper limit of 100 per cent as well as a lower limit of zero. These bounds can be used, together with the median (unless the median is 0 per cent), to fix a geometric progression which covers not the actual, but the potential range of the data (0 to 100 per cent). For a given number of classes, the lower the median, the higher the base to be used, as is more appropriate for an increasingly J-shaped frequency distribution.

If there are N classes in the progression to base x, and therefore $2N$ in the progression to base $x^{\frac{1}{2}}$, both of which go from 0 to 100, equation (2) gives:

$$100 = b \sum_{c=0}^{c=2N-1} x^{c/2} = \frac{a}{1+x^{\frac{1}{2}}} \sum_{c=0}^{c=2N-1} x^{c/2} = \frac{a}{1+x^{\frac{1}{2}}} \frac{1-x^N}{1-x^{\frac{1}{2}}} = \frac{a(1-x^N)}{1-x} \qquad (11)$$

combined with equation (7), we have:

$$P = \frac{100 \sum_{c=0}^{c=N-1} x^{c/2}}{\sum_{c=0}^{c=2N-1} x^{c/2}} \qquad (12)$$

This is equivalent (A. Young and R. Gawley, pers. commun.) to the quadratic in $x^{N/2}$;

$$P\,x^N - 100\,x^{N/2} + 100 - P = 0 \qquad (13)$$

with roots of $x = 1$ and $x = (100\text{-}P)/P$

Hence the appropriate base can be calculated directly from the median as

$$x = \left(\frac{100-P}{P}\right)^{2/N} \qquad (14)$$

This is easily calculated, but by way of exemplification Table IX gives base, x, as a function of (observed) median and of number of classes. Given x, the size of the initial class can be calculated from equation (10), and further class limits from equation (6). For example, if the median of a variable which cannot exceed 100 per cent is 7·8 per cent, the appropriate base which gives five classes is 2·686 (from Table IX), the first class is $7{\cdot}8(1-2{\cdot}686)/(1-2{\cdot}686^{2\cdot5})$ = 1·21 97 (from equation 10), and the class boundaries are 0, 1·215, 4·478, 13·244, 36·788, 100·028 (from equation 6).

If rounding errors give an uppermost class limit, q, slightly different from 100 per cent, this can be corrected by rescaling each limit by $100/q$. It is best not to interpolate in this Table, but to use the equations and carry a sufficient number of decimal places.

Alternative systems for closed percentages involve equal intervals on trigonometric transformations of the percentage scale, for example the arcsin (square root). If the 100 per cent range is covered, the latter gives limits which are symmetrical about 50 per cent; for five classes, the limits are 0·00, 9·55, 34·55, 65·45, 90·45, and 100·00 per cent. These must *not* be multiplied by a constant.

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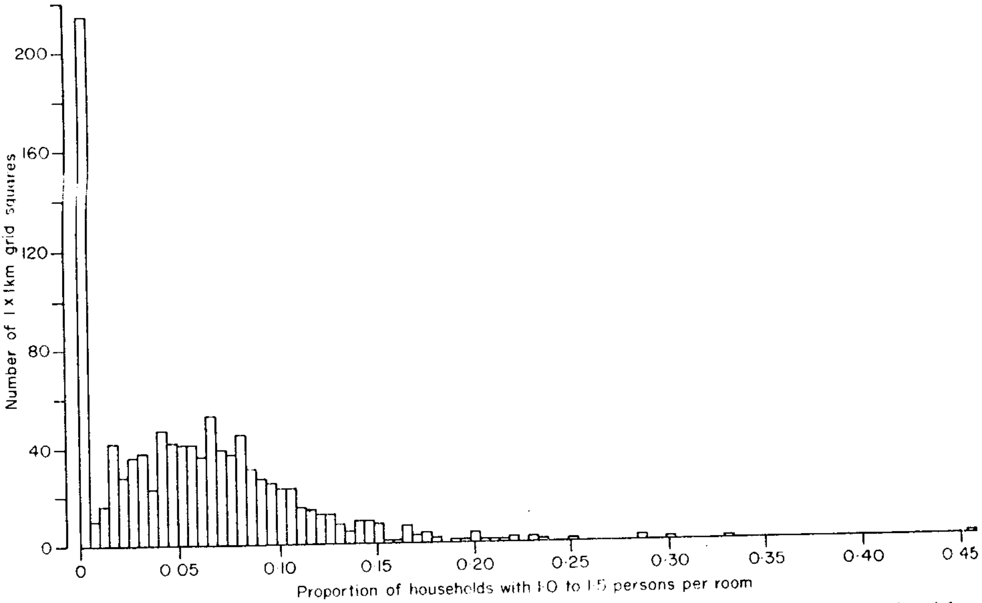

FIGURE 3 Histogram of proportion of households with 1 to 1·5 persons per room, for 1043 grid squares with eight or more households in County Durham, 1971: a frequency distribution with a secondary mode at zero

There are, in fact, theoretical grounds for applying the arcsin (square root), otherwise known s the angular, transformation to closed percentages. If the denominators vary little, this trans-ormation stabilizes the variance of ratios whose numerators follow a binomial distribution. Simultaneously it reduces the positive skew of percentages with low means and the negative kew of closed percentages with high means (approaching 100 per cent or 90°), while having ittle effect on those with means near 50 per cent (45°), which are usually unskewed. In reality, he variability of denominators complicates matters, as do other spatial heterogeneities, but near-normality and fairly constant variance is achieved for the closed percentage variables of age structure, household size and crowding taken from the 1 km grid-square census data for Great Britain, 1971. Hence the angular transformation permits standard-deviation class intervals to be applied to many census variables. It is not successful for birthplace variables which have very low mean percentages and remain skewed even after transformation: for these, geometric pro-gressions are required, even though they are not perfect and leave several classes underutilized. Likewise, for Great Britain, the percentages of population born in England, in Wales and in Scotland are bimodal due to mixing of squares for the country involved with squares for the other two countries; their standard deviations are excessively high relative to their means, and equal class intervals are therefore used, as is also the case for some tenure variables.

MODES AT ZERO

Frequency distributions often have a secondary mode at zero (Fig. 3), making them significantly bimodal. Standard deviation or range-based classes may pick out the bimodality because the second lowest class may occur less often than those on either side. But the most elegant solution, if zero is a frequent occurrence, is probably to treat zero as one class, separately, and to subdivide the remainder according to a precise system based on their frequency distribution. This is especially useful when the median is zero.

Truly bimodal distributions may be mapped on either a range or a standard-deviation basis, with some loss of mapping efficiency in that several classes may be of rare occurrence. If a small number of classes is used, it is necessary to check that the incidence of class limits does not disguise the bimodality; in this rare case, as suggested by the proponents of 'natural breaks', it is best that one or more class boundaries should fall in the trough between the two modes.

CONCLUSION

In summary, a standard-deviation-based class interval with open highest and lowest classes is of widest applicability, for all variables which can be transformed to essentially unimodal, symmetrical frequency distributions. For rectangular distributions, range division is best, while for J-shaped distributions a geometric progression of class width is chosen. This is the combination used for the maps in Dewdney and Rhind (1976), where a class size of one standard deviation was most often used. In each case, information about the frequency distribution should be implicit in the map. Hence it is necessary that careful analysis of frequency distributions should precede mapping, and indeed precede any quantitative analysis.

Percentile-based classes or nested-mean limits may perhaps facilitate comparison of maps of variables with different types of frequency distribution, but they must be interpreted very carefully in relation to the frequency distribution. Exogenous class boundaries provide useful reference points, if available, but idiographic boundaries should almost never be used. These considerations apply most strongly when a small number of classes is to be used. For proportional symbolization, the 'intervals' used are effectively equal, preferably on a measurement scale which gives a symmetrical frequency distribution.

To discuss class intervals solely in the context of choropleth maps is misleading; class intervals are required for any type of graded symbolization, based on point, line or area symbols. Such grading or classification is necessary if the class of individual symbols is to be accurately perceived, but not if the aim is to present a synoptic, photograph-like picture of a distribution (Tobler, 1973). However, technical limitations in both automated and manual cartography often necessitate definition of a finite number of classes.

ACKNOWLEDGEMENTS

I am very grateful to David Rhind, Nick Cox and Roger Gawley for their help and encouragement, and to Joan Dresser for typing the manuscript. This study forms part of the work of the Census Research Unit, funded by the U.K. Social Science Research Council; data were supplied by the Office of Population Censuses and Surveys.

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