



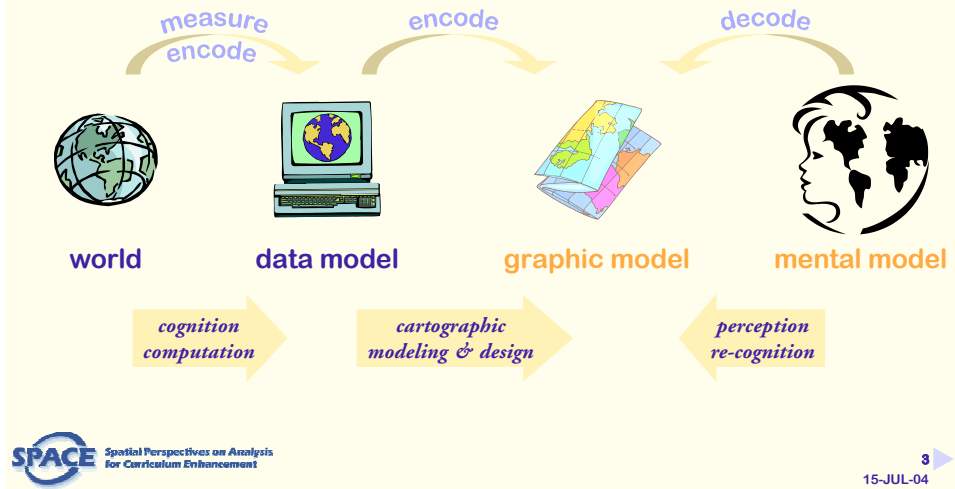
## statistical mapping

volumetric data

### outline

- **volumetric data**
  - areas: choropleth
- **classification**
  - to class or not to class?
  - evaluate classification solution
- **design issues**
  - legend
  - color

## cartographic process

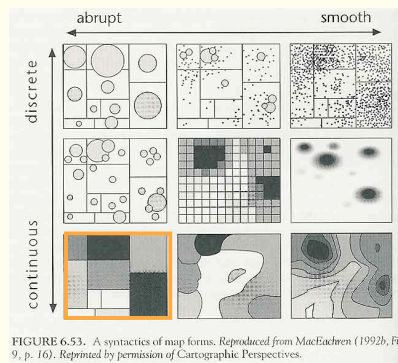


## data model vs. graphic model

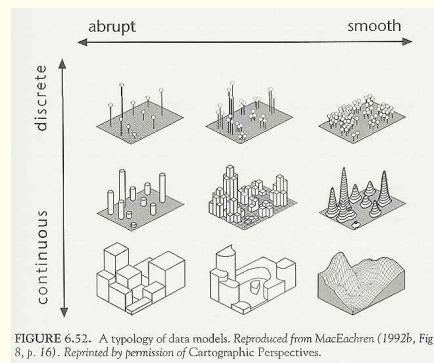
goal: graphic model vs. data model → best fit

- data model: volumes (continuous, 3D) at points & areas

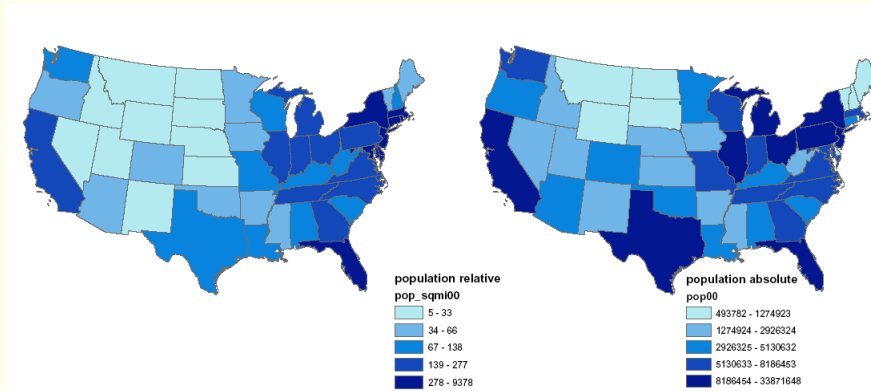
2D gm



3D gm

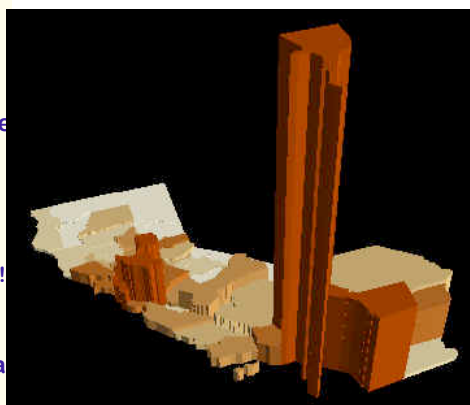


## U.S. population in 2000 (volumes)



## volumetric data in areas

- choropleth map
  - choros = place, space
  - plethos = magnitude
- continuous data: ratios, densities
- discrete graphic model
  - stepped surface
  - boundaries unrelated to data
  - adjust data model: standardize!
- good for...
  - finding value of a given area
  - gist of overall pattern
  - compare patterns between n



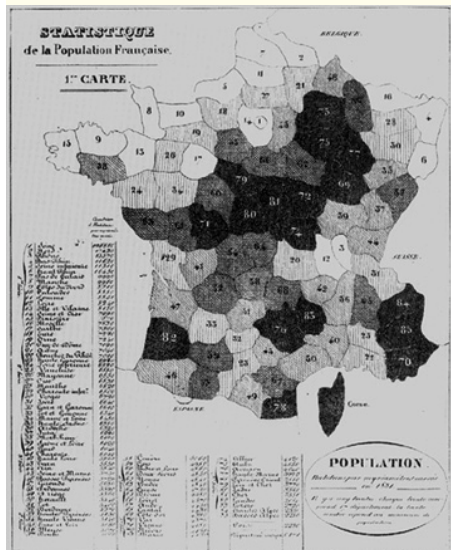
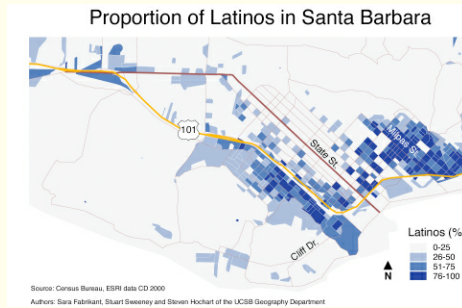
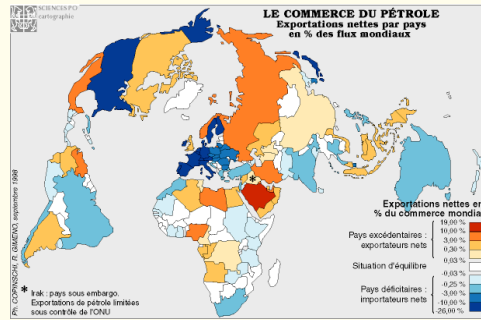
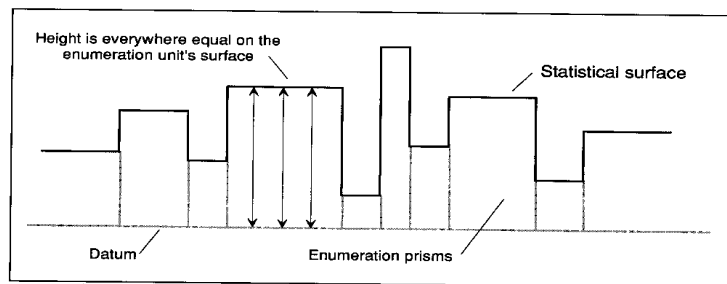


Figure 52 D'Angeville's 1836 map of the number of persons per square myriametre in France. Original 187 x 239 mm. Lithograph. (Photo. Bibliothèque Nationale, Paris.)  
source: D'Angeville, 1836, in: Robinson

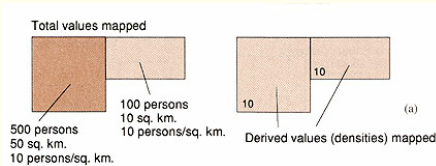


## discrete statistical surface model



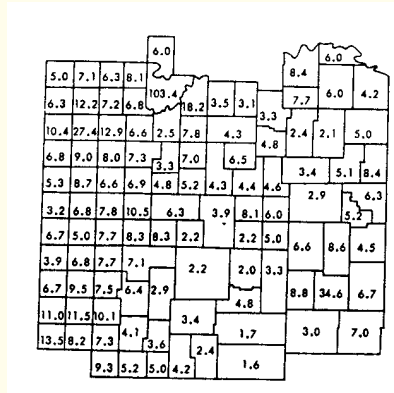
Source: Robinson

- do NOT use raw data
- but a variable per unit per area



Source: Dent

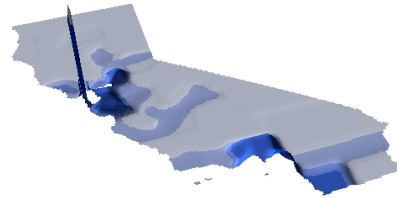
## raw map data → 3D symbolization space



Source: Robinson

## graphic representations of map data

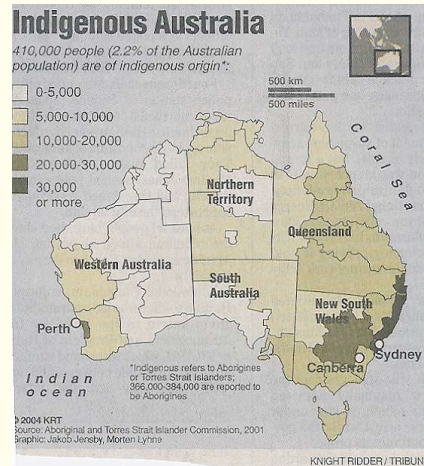
- 3D representations
- discrete surface maps
  - pin head maps
  - 3D bar maps
  - prism maps
- continuous surface maps
  - e.g. using centroid of unit area
    - various interpolation methods
  - e.g. using boundary unit area
    - pycnophylactic interpolation



sara fabrikant, 2001

## outline

- volumetric data
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- classification
  - to class or not to class
  - evaluate classification solution
- design issues
  - legend
  - color



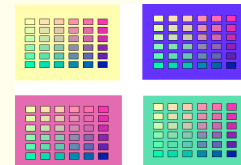
## choropleth map types

### classed choropleth maps

- data more aggregated (stat. error ↑)
- perceptual limits of how many categories can be perceived
  - not more than 11 area-shaded gray tones
- 5-7 classes appropriate most of the time (perc. error ↓)
  - (Miller:  $7 \pm 2$ )
- if animated, closer to 3 classes

### unclassed choropleth maps

- data less aggregated (stat. error ↓)
- many individual values (perc. error ↑)  
(based on empirical findings)



## unclassed choropleth map

pro: (e.g., Tobler)

- stat. data = continuous
- data model = graphic model

con (e.g., Dobson)

- graphic model <> mental model
- map <> legend
- distribution dependent
- map comparison is hard

solution:

- ESDA: make both types!

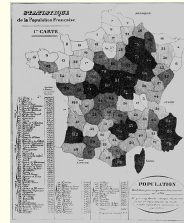
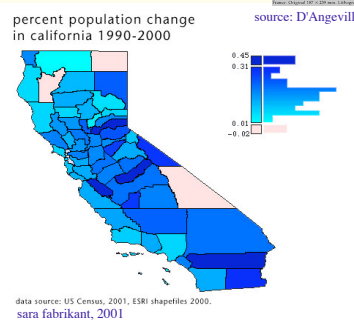


Figure 10.7: D'Angeville's 1836 map of the number of persons per square mile in the counties of California. (Source: Robinson, 1995, p. 107.)



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## to class or not to class?

classing most useful if distribution (Evans, 1977)...

- shows natural breaks
- is multimodal
- is in some progression
- of phenomena show concrete breaks or distinctions
  - (e.g., people, buildings etc.)
- classing is useful because this is how the brain works
  - categorization

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## classification components

- how many classes?
- what partitioning scheme?
- evaluate error pattern introduced by partitioning?

## how many classes?

it depends!

- map audience
- spatial pattern of phenomenon

optimization problem

- fewer classes to decrease map complexity
- fewer classes to improve legibility
- more classes to reduce classification error/generalization
- more classes to show more information/ “more truth”



## what partitioning scheme?

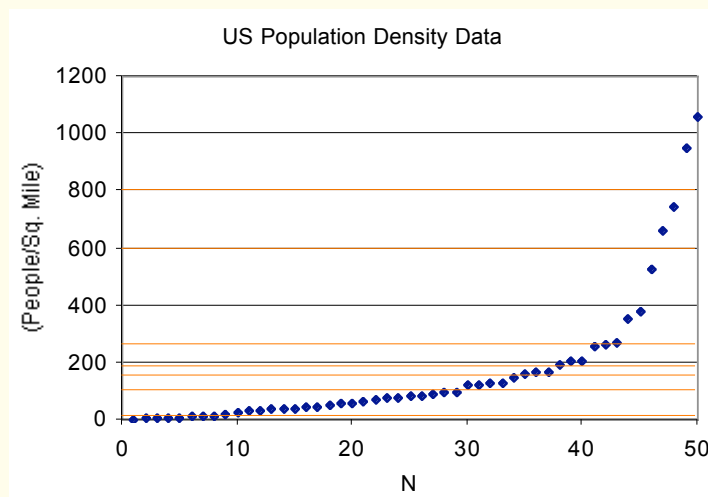
guess what: it depends!

- **idiographic** classes based on the nature of the data?
  - e.g. look for natural patterns in data
- **arbitrary** classes with round numbers?
  - e.g. 10-20, 21-40, etc.
- **serial** classes based on mathematical principles?
  - e.g. geometric progression
- **exogenous** classes based on a related variable?
  - e.g. income based on a “poverty” variable



recipe...

- start by creating a graph of your data (lab: part I !)
- rank your data: plot lowest to highest values



## ideographic schemes

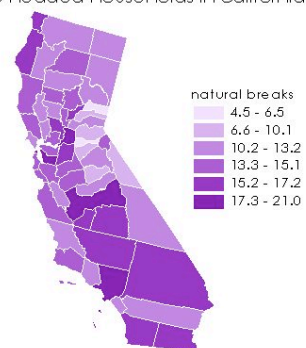
Greek - “descriptor of uniqueness”

- **clinographic - natural breaks**
  - look for discontinuities in array (data unevenly distributed)
- **quantiles based (n-tiles)**
  - data values evenly segmented (data evenly distributed, compare ranks)
- **contiguous**
  - spatially homogeneous (data spatially correlated)
- **correlation based**
  - high similarity (data semantically correlated)

## natural breaks

example...

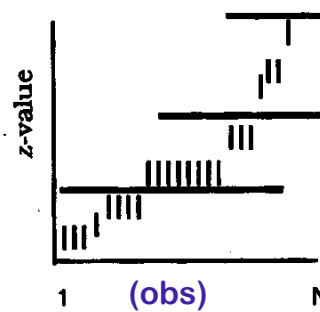
female headed households in california counties in 1999



sources:  
boundary files: ESRI ArcView 3.2 data C.D  
attribute data: US Bureau of the Census

0 50 100 150 miles

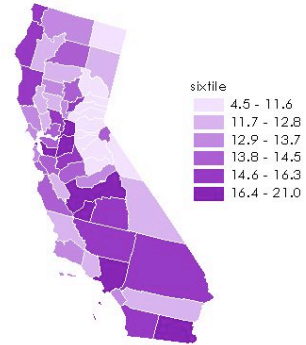
look for gaps in  
the array of values (y-axis)



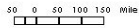
## quantiles

### example...

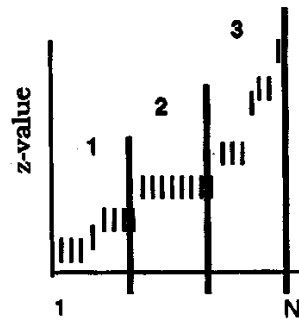
female headed households in california counties in 1999



sources:  
boundary files: ESRI ArcView 3.2 data CD  
attribute data: US bureau of the Census



put equal number of observations in each class (x axis)



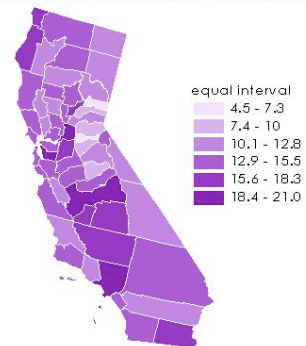
21

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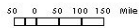
## equal interval

### example...

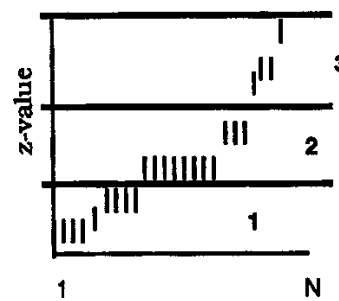
female headed households in california counties in 1999



sources:  
boundary files: ESRI ArcView 3.2 data CD  
attribute data: US bureau of the Census



put equal value range in each class



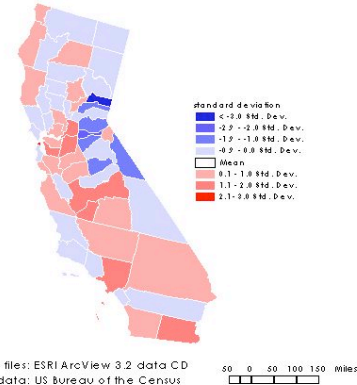
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## standard deviation

### example...

female headed households in california counties in 1999



sources:  
boundary files: ESRI ArcView 3.2 data CD  
attribute data: US bureau of the Census

average of the absolute deviations from the mean

all distributions  
 $m \pm 2s = 75\% \text{ obs.}$   
 $m \pm 3s = 80\% \text{ obs.}$

normal distribution  
 $m \pm 1s = 68\% \text{ obs.}$   
 $m \pm 2s = 95\% \text{ obs.}$   
 $m \pm 3s = 99\% \text{ obs.}$

## optimization method

the problem:

- for 10 values (n) and 3 groups (r) there are 36 ways to contiguously partition the data!

$$\frac{(n - 1)!}{(r - 1)! \left[ (n - 1) \square (r - 1) \right]!}$$

goal:

find exhaustive set of distinctly different groups, while keeping groups internally most homogeneous  
 → optimization problem (e.g., cluster analysis)!

## what class breaks?

class breaks such that...

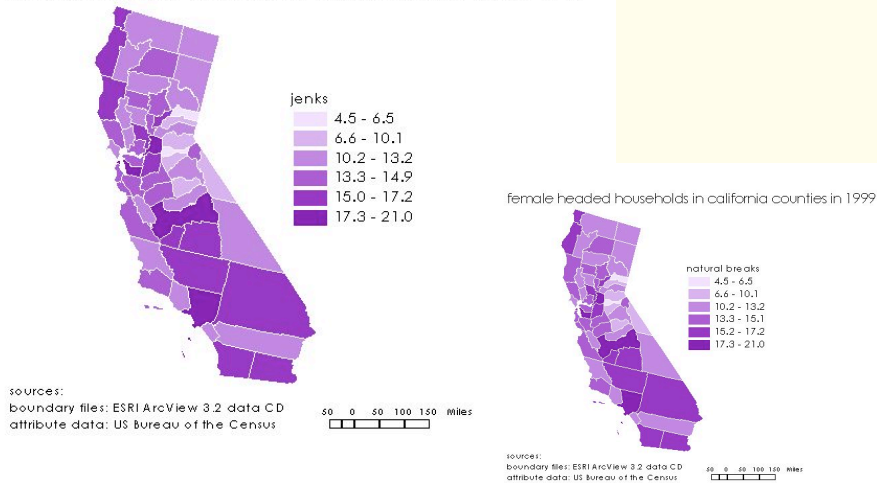
- **similarity within** the classes is maximal, &
- **differences between** the class is maximal.
  - natural breaks (-> where large gaps occur)
- **iterative mathematical procedure**
  - find a global minimum of within-class dispersion, and find a global maximum of between-class dispersion
- **least squared distance from the (class) mean**
  - minimize the blanket of error:
  - smallest sum of weighted squared deviations from the class mean

$$\text{Dist.}_{\min} = \sum_{i=1}^N w_i (z_i - \bar{z}_i)^2$$

area obs<sub>i</sub>                      j=1

## optimized solution

female headed households in california counties in 1999



## goodness of variance fit - GVF

Jenks (1977)

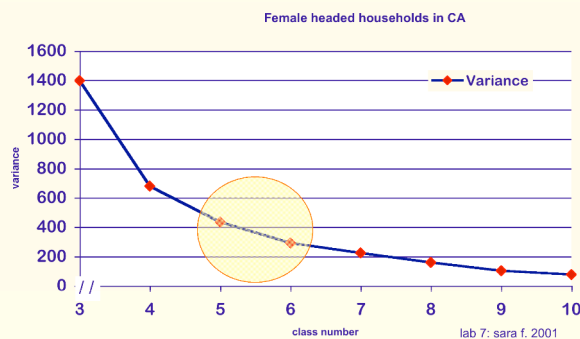
- second step to maximize distance between classes

**GVF** = sum of squared deviations between classes

- compute squared deviations from overall mean (data array) and subtract squared deviations from the class means
- go through various iterations until GVF is maximized
- Range of GVF
  - max. 1.0 = each value is one class → unclassified choropleth map
  - min. 0.0 = one class only
  - ideal: closest to one as possible

## how many classes?

- minimize number of classes without losing too much information
- find **elbow** in the variance graph...



## evaluation of partitioning method

Jenks and Coulson (1963)

- visual check on partitioning validity
  - remember choropleth data model: a statistical surface
- compute discrepancy between each value and its associated class mean

### Blanket of Error

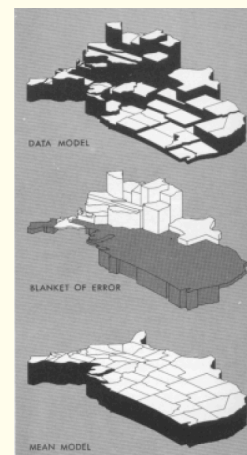
- the classification error = statistical surface (the error values fluctuating above and below the class mean)
- akin to root mean square error (RMSE)  
e.g., standard deviation from mean

$$\text{RMSE} = \sqrt{\frac{\sum (x_i - \bar{X})^2}{N}}$$

## blanket of error

Jenks (1967):

*“We have found in our study, however, that the series of classes with minimal error and those with an uniformly distributed error are not significantly different statistically. As a result we assume that the cartographer should use equal average or equal relative deviation classes for choroplethic maps.”*

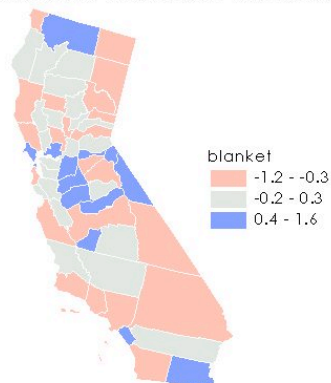


## evaluation (cont.)

- what is the error pattern on the map?
  - is the blanket of error uniformly distributed?
  - is concentrated in certain areas and not in others?
  - if yes, why?
- error measure is sensitive to # of classes and classing scheme!
- with optimized classing, the error should be minimized
  - Natural Break (Jenk's) method in ArcMap is minimizing
- cartographer controls this error by modifying classing scheme!

## blanket of error (cont.)

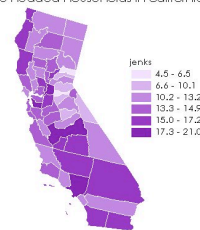
female headed households in california counties in 1999



sources:  
boundary files: ESRI ArcView 3.2 data CD  
attribute data: US Bureau of the Census

50 0 50 100 150 Miles

female headed households in california counties in 1999

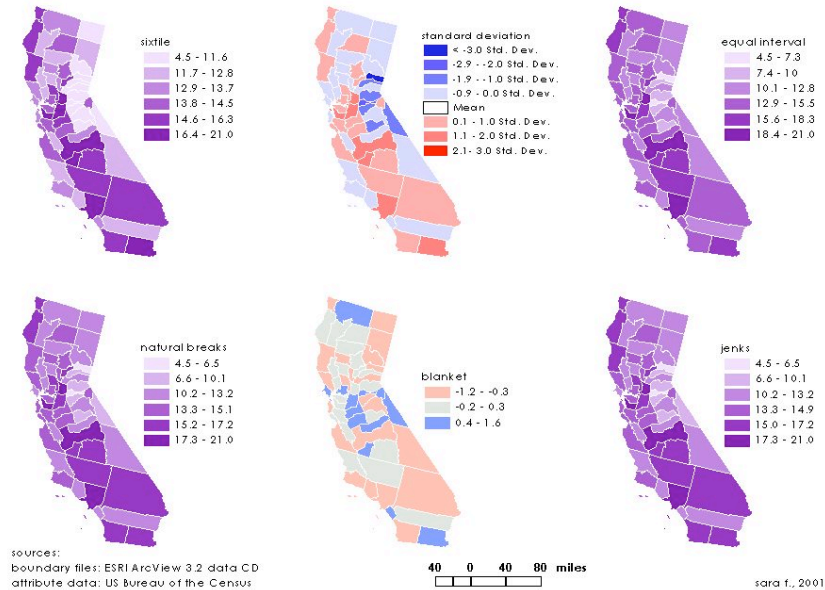


sources:  
boundary files: ESRI ArcView 3.2 data CD  
attribute data: US Bureau of the Census

0 20 40 60 80 100 Miles



female headed households in california counties in 1999  
**identical data, same pattern?**



## classed choropleth maps

- summary: many possible schemes !

### which solution is best?

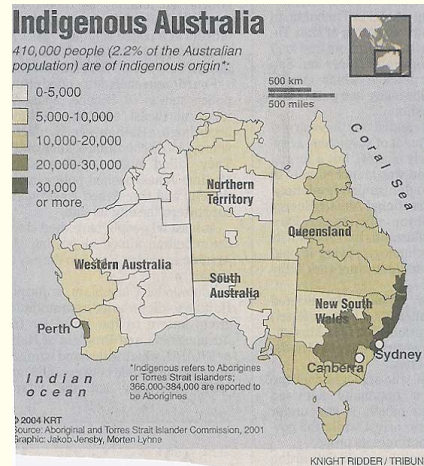
- it depends on the data (always inspect distribution!)
- it depends on the scale (ecological fallacy, MAUP)
- several good solutions possible!

### data requirements

- based on enumeration unit (e.g. census tract)
- no totals, as enumeration units vary in size!
- standardized data (ratios of some sort)
  - density (area dependent)
  - per capita income (area independent)

## outline

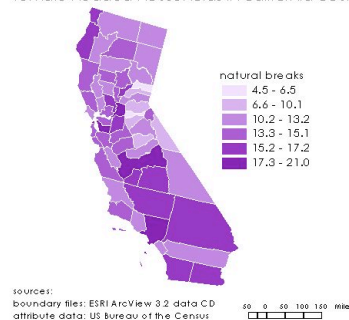
- volumetric data
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## legend design: an important side note...

- map and legend on the same page
- correct labeling of classes
  - simple, concise and legible
- beware of software 'defaults'
  - e.g. fix ESRI-isms
- all values have to be classed
  - no gaps, no overlapping bins
- example...
  - 0 - 10 can be labeled: < 11
  - 11 - 20
  - 21 - 40
  - 41 - 60 can be labeled: > 40

female-headed households in California counties in 1999



## color guidelines

- visual variables hue and value applied to choropleth maps

Cindy Brewer (Penn State):

*“Appropriate use of color for data display allows interrelationships and patterns within data to be easily observed. The careless use of color will obscure these patterns.”*

guidelines based on...

- carto/graphic experience (art)
- empirical studies (science)

