Geographical Movement

is critically important.

This is because much **change** in the world is due to geographical movement.

Movement of people, ideas, money, or materiel.

The Table is an Important Form of Geographic Movement Data

Especially when the rows and columns refer to known geographic locations.

The tables are then “square”, having the same number of rows as columns.

Such tables can be decomposed into two parts, a symmetric part and a skew symmetric part.

For the statisticians in the audience the total variance can also be partitioned into these two parts.

An example of an asymmetric geographical table.

Polynesian Communication Charges ($)

To From	CI	Fiji	F P	Ki	N C	PNG	SI	To	Tu	Va	WS	Aust	Fr	Ja	NZ	UK	USA
Cook I		5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	9.35	9.35	3.70	9.35	9.35
Fiji	3.29		3.50	3.29	3.50	3.29	3.29	3.29	na	3.50	3.29	3.29	5.55	5.55	3.29	5.55	5.55
F Polynesia	14.85	14.85		na	7.95	14.85	14.85	14.85	na	7.95	14.85	12.73	15.91	24.39	12.73	21.21	24.39
Kiribati	6.50	6.50	6.50		6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	9.28	9.28	6.50	9.28	9.28
N Caledonia	13.85	13.85	7.79	na		11.70	11.70	13.85	na	7.79	11.70	11.70	15.61	17.53	11.70	19.91	23.36
Papua N G	5.67	5.67	5.67	5.67	5.67		5.67	5.67	5.67	5.67	5.67	5.67	11.33	11.33	5.67	7.55	11.33
Solomon I	5.30	4.12	5.30	5.30	5.30	4.12		5.30	5.30	5.30	5.30	3.82	8.53	8.53	4.12	8.53	8.53
Tonga	3.47	3.47	3.47	3.47	3.47	3.47	3.47		3.47	3.47	3.47	3.47	6.95	6.95	3.47	6.95	6.95
Tuvalu	5.80	4.64	5.80	5.80	5.80	5.80	5.80	5.80		5.80	5.80	3.48	9.28	9.28	9.28	9.28	9.28
Vanuatu	7.96	5.24	7.96	7.96	5.24	7.96	7.96	7.96	7.96		7.96	5.24	10.29	10.29	5.24	10.29	13.02
W Samoa	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86		3.86	5.14	5.14	3.86	5.14	5.14
Australia	3.71	3.71	3.71	3.71	3.71	3.02	3.02	4.87	na	3.71	3.71		3.71	4.87	3.02	3.71	3.71
France	12.80	12.80	7.53	na	7.53	12.80	12.80	12.80	na	12.80	12.80	11.17		11.17	11.17	2.63	5.47
Japan	6.27	6.27	6.27	6.27	6.27	6.27	6.27	6.27	na	6.27	6.27	6.27	7.88		6.27	7.88	5.20
N Zealand	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	5.40	5.40		5.40	5.40
UK	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	5.84	2.66	8.21	5.84		4.11
USA	5.77	5.77	5.49	5.77	5.77	5.77	5.77	5.72	5.77	5.77	5.72	4.66	5.27	4.14	5.53	4.08	

R.G. Ward, 1995, “The Shape of the Tele-Cost Worlds”, A. Cliff, et al, eds., *Diffusing Geography*, p. 228.

Let T represent the table, with i rows and j columns. It can be decomposed into two parts as follows:

$$T_{ij} = T^{+} + T^{-}$$

where

$$T^{+} = (T_{ij} + T_{ji})/2 \quad \text{(symmetric)}$$

$$T^{-} = (T_{ij} - T_{ji})/2 \quad \text{(skew symmetric)}$$

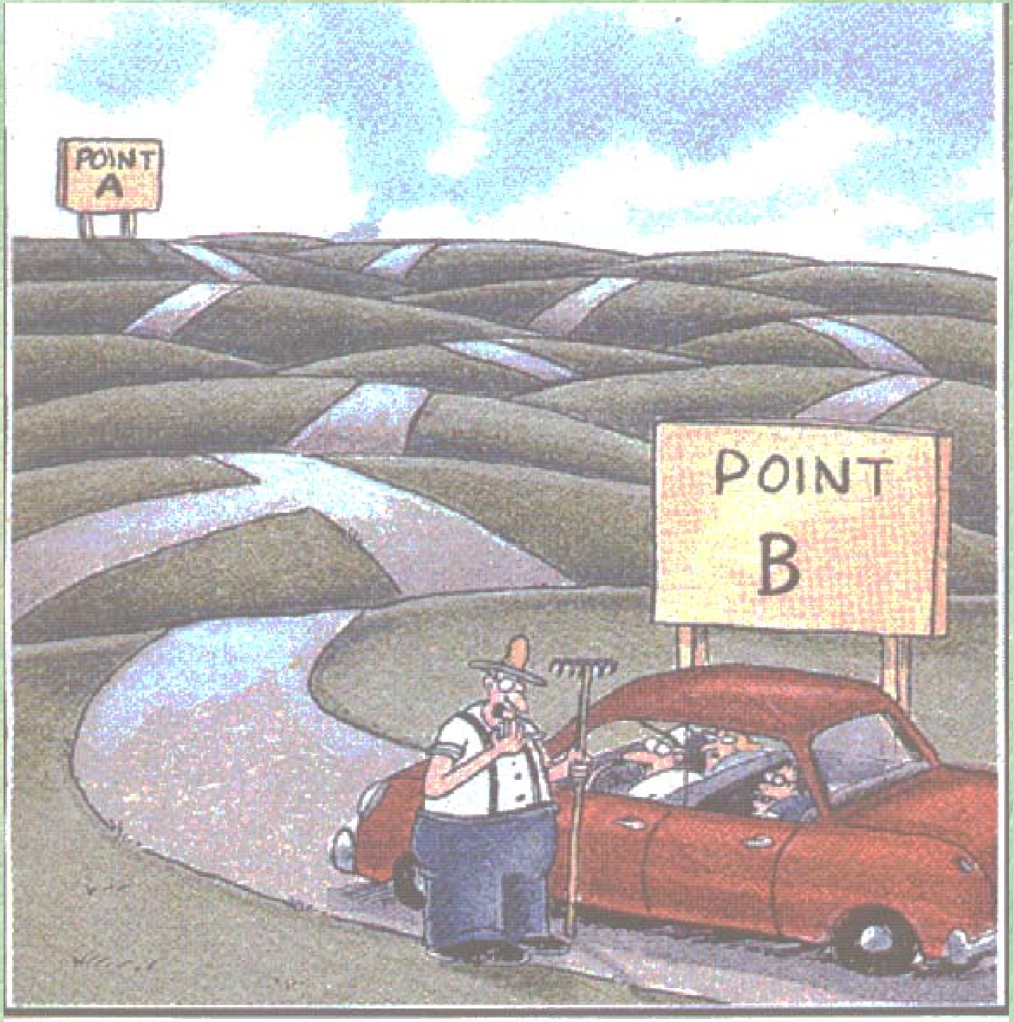
Asymmetries Are a Fact of Geography

Movement is generally not symmetric.

This gives rise to asymmetries which can be exploited.

From B to A is Not the Same as A to B

(Gary Larson)



"Well, lemme think. ... You've stumped me, son. Most folks only wanna know how to go the other way."

Asymmetrical Tables Can Lead To The Construction Of A Vector Field

Start with an asymmetrical geographical table. There are many such tables!

It is possible to compute the degree of asymmetry for such tables, and to partition the total variance into symmetric and skew symmetric variances

To construct the vector field it is necessary to know the geographic locations and to invoke a model of the process.

Both Parts Can Be Used

When geographic locations are not known the symmetric part can be used to estimate the positions using a two dimensional trilateration (a.k.a. multidimensional scaling).

The skew symmetric (asymmetric) part can be used to infer movement.

A simple example

Table of Mail Delivery Times

Observe the asymmetry

Transit time for US mail, in days (1973)

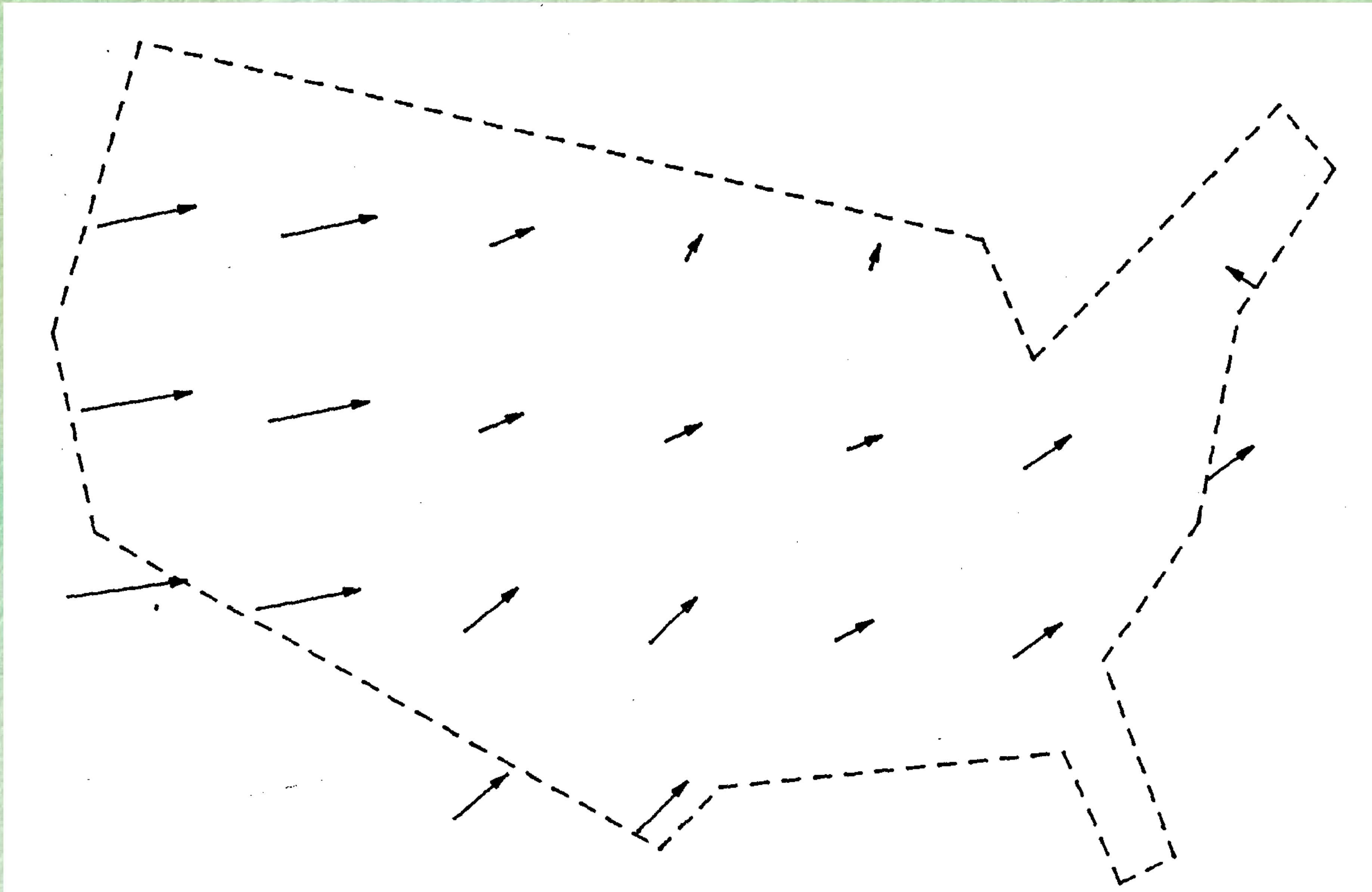
From: \ To:	NYC	CHI	LAX	WDC	STL	HOU
NYC	0.9	1.8	2.5	2.0	2.3	2.3
CHI	2.6	0.8	3.1	2.2	1.9	2.3
LAX	2.5	2.2	1.1	2.2	2.3	2.6
WDC	1.8	2.3	2.6	1.3	2.4	2.5
STL	2.4	2.1	3.1	2.4	0.9	2.5
HOU	2.3	1.9	2.8	2.2	2.2	1.1

The table of mail times is asymmetric

Its degree of asymmetry is 66%

The asymmetry is used to compute a map

A Map of Wind Computed from Mail Delivery Times



From Wind to Pressure Field

An interesting property of vector fields, as on the foregoing map, is that they may be inverted.

If you think of a vector field as having been derived from the topography of some surface this assertion is that the topography can be calculated when only the slope is known.

At least up to a constant of integration (the absolute elevation) and if the data are curl free.

In the particular instance here, this says that the barometric pressure could be estimated from the mail delivery times.

Another example

In the United States the Currency Indicates Where It Was Issued

For bills this is the Federal Reserve District.

Coins contain a mint abbreviation.

You can check your wallet to estimate your interaction with the rest of the country.

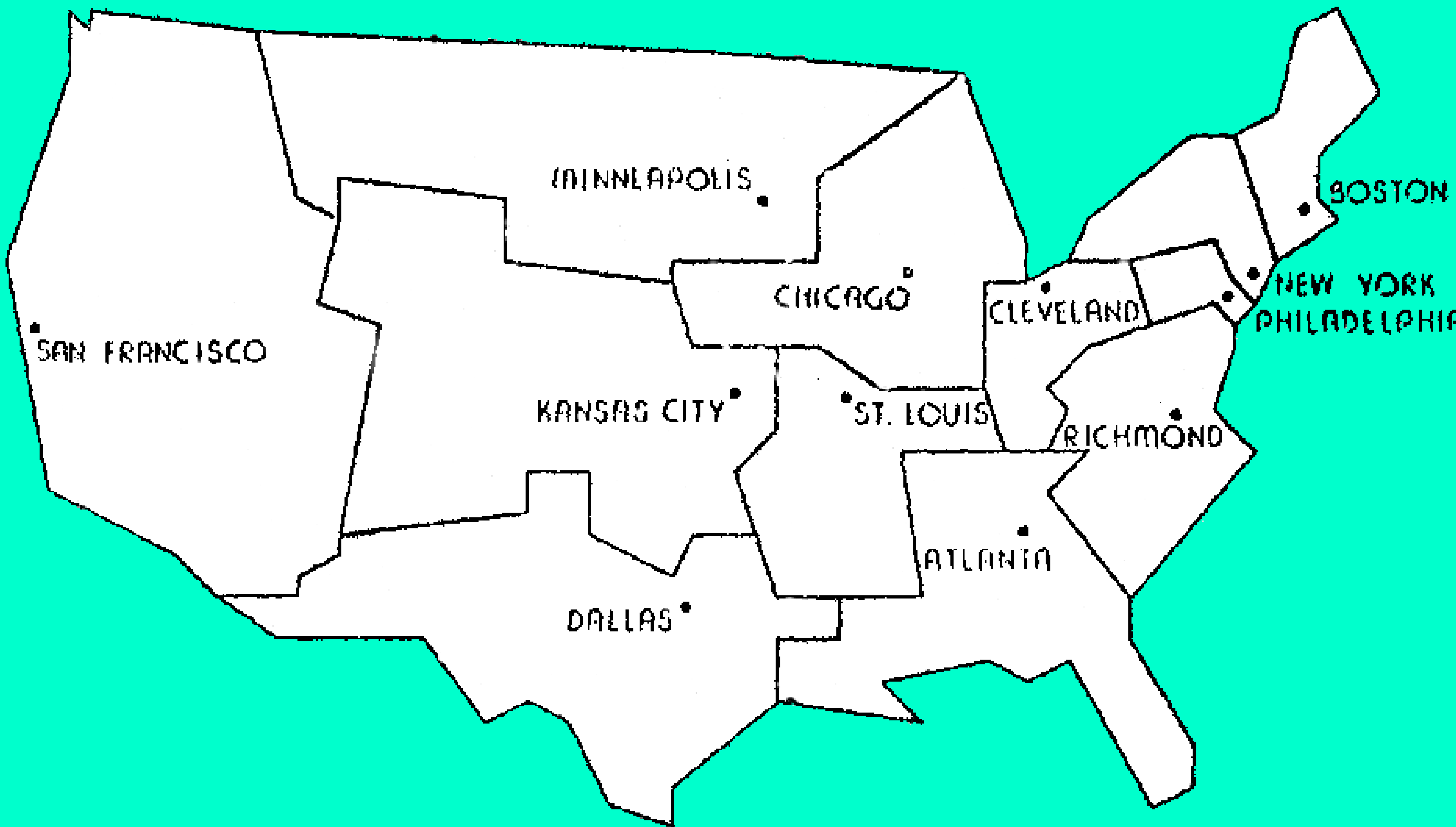
Dollar Bill
(Federal Reserve Note)

Issued by the 8th (St. Louis) Federal Reserve District.

(H is the 8th letter of the alphabet)

The 12 Federal Reserve Districts

(Alaska and Hawaii omitted)



Movement of One Dollar Notes

between Federal Reserve Districts, in hundreds, Feb. 1976

From: \ To:	B	NY	P	Cl	R	A	Ch	SL	M	K	D	SF
Boston	2040	289	47	52	137	118	90	10	16	15	13	138
New York	602	1980	231	209	388	307	286	15	48	26	18	261
Philadelphia	143	414	860	84	342	130	134	8	25	10	10	80
Cleveland	68	192	47	1296	171	177	618	16	44	43	19	131
Richmond	150	266	158	226	3899	578	295	20	62	54	22	152
Atlanta	122	159	57	186	319	3741	439	30	51	78	102	189
Chicago	97	155	39	496	143	266	5630	74	278	100	40	290
St. Louis	31	56	14	142	80	201	573	342	46	128	47	109
Minneapolis	14	26	11	32	29	41	295	10	1438	51	14	138
Kansas City	20	41	8	55	40	71	215	33	120	811	86	247
Dallas	31	41	8	38	46	165	125	20	37	253	788	203
San Francisco	82	81	23	84	114	106	251	22	127	128	43	5380

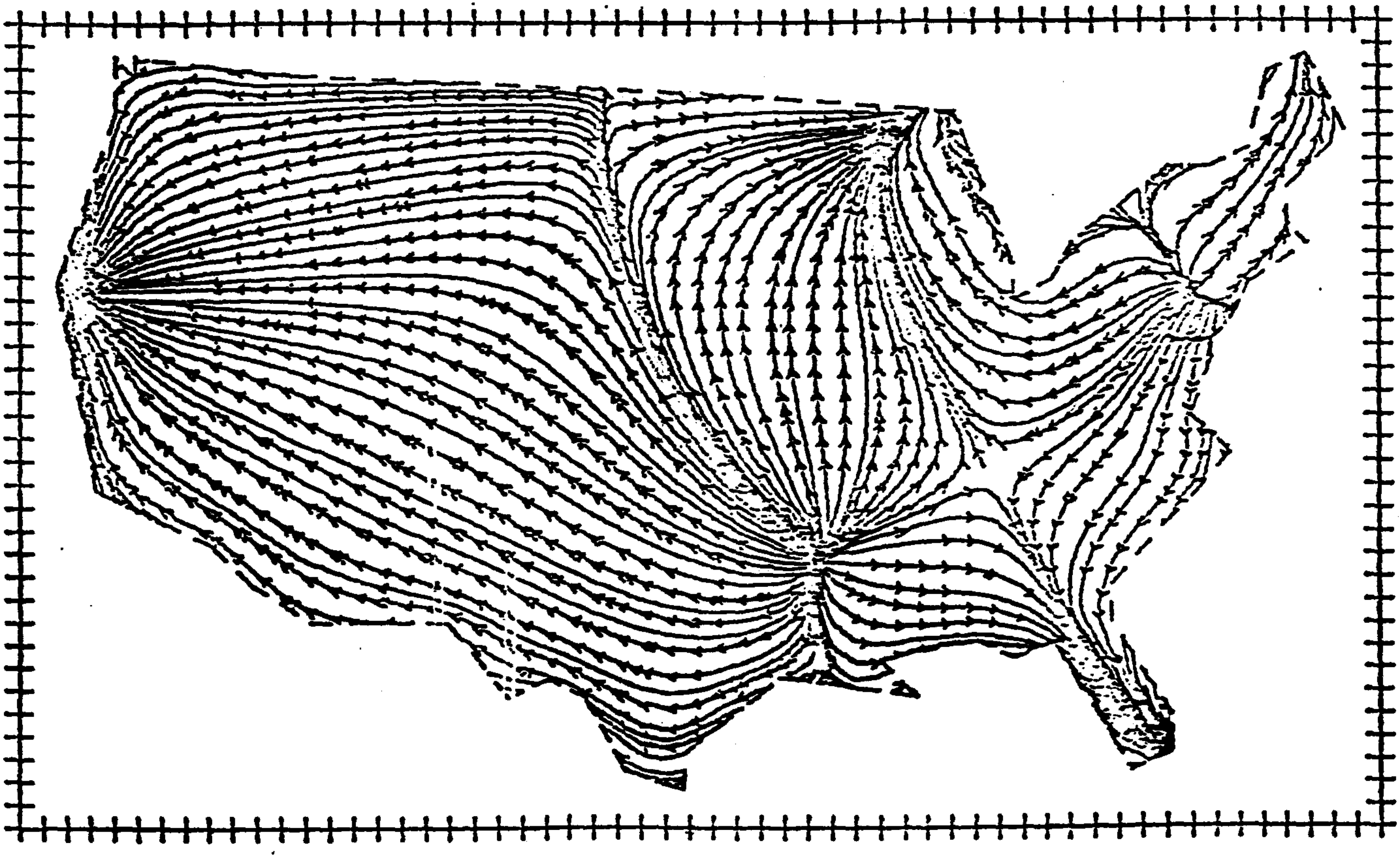
The Table of Dollar Bill Movements

was obtained from MacDonalds outlets throughout the United States.

Source: S. Pignatello, 1977, *Mathematical Modeling for Management of the Quality of Circulating Currency*, Federal Reserve Bank, Philadelphia

From the table we can compute a movement map.

Dollar Bill Movement in the U.S.



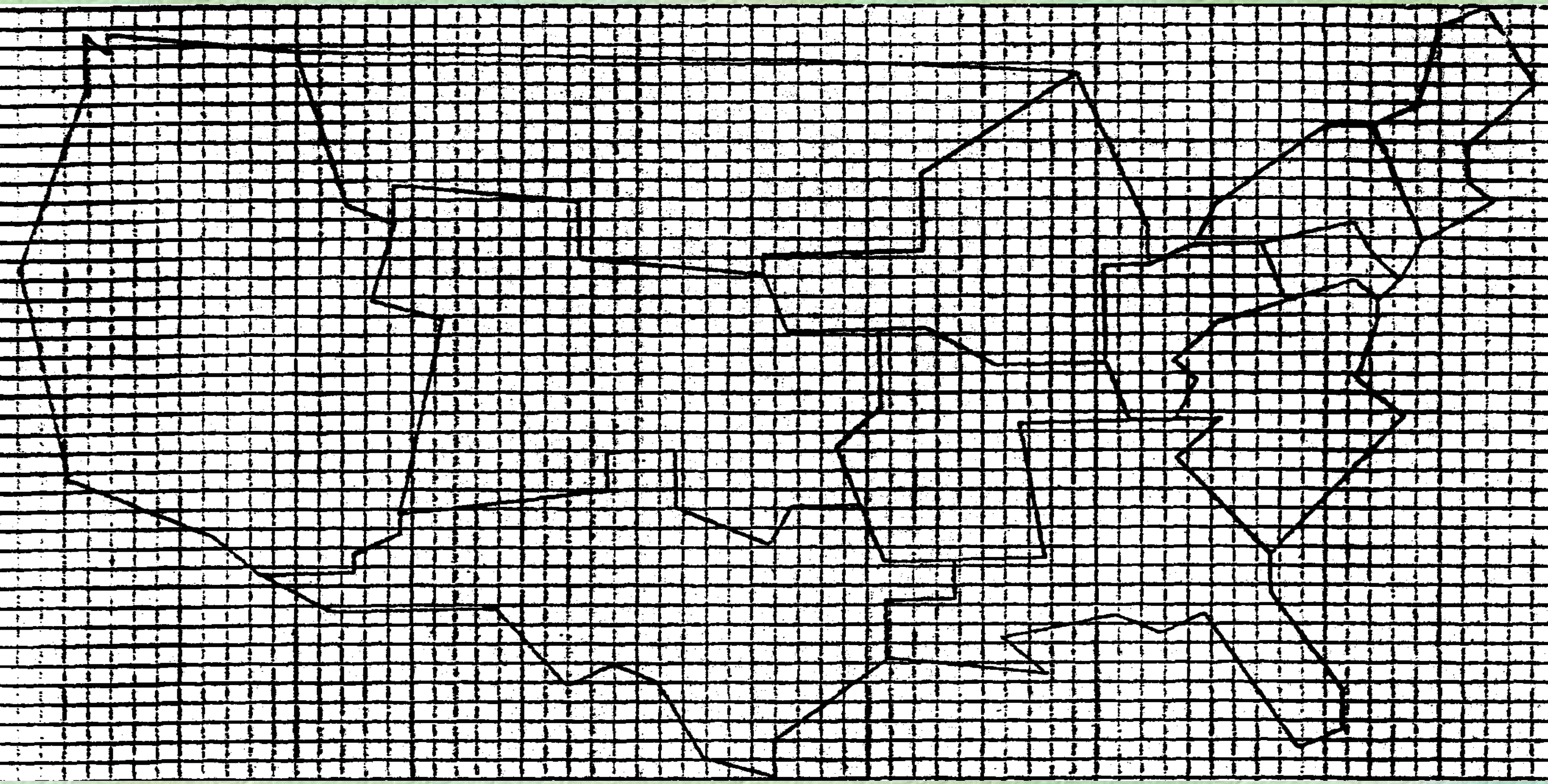
The Map is Computed Using a Continuous Version of the Gravity Model

The result is a system of partial differential equations solved by a finite difference iteration to obtain the potential field.

This can be contoured and its gradient computed and drawn on a map.

W. Tobler, 1981,"A Model of Geographic Movement", *Geogr. Analysis*, 13 (1): 1-20
G. Dorigo, & Tobler, W., 1983, “Push Pull Migration Laws”, *Annals*, AAG, 73(1):1-17.

First the Federal Reserve Districts Are "Rasterized"

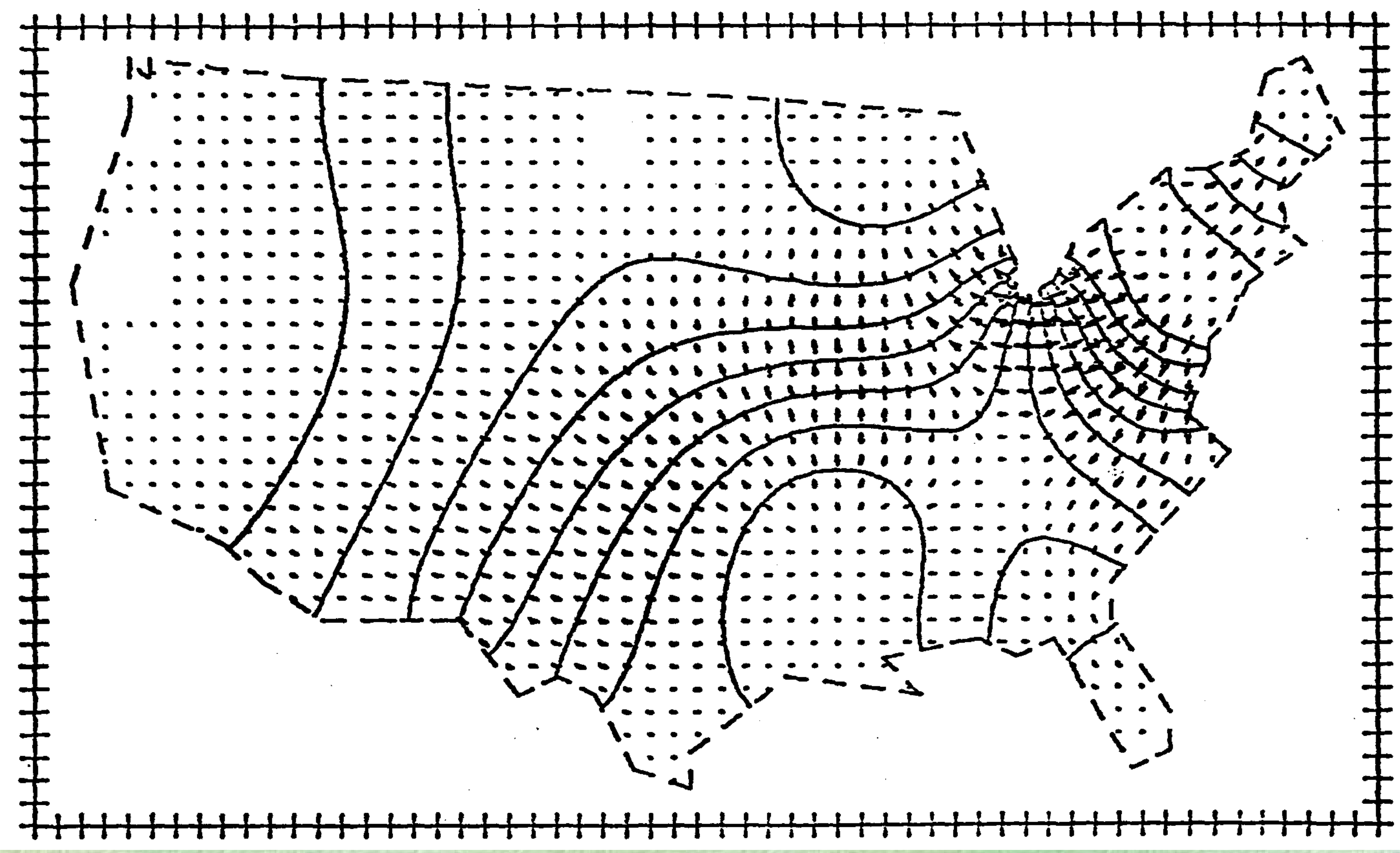


There will be one finite difference equation for each node on this raster

(2088 simultaneous equations)

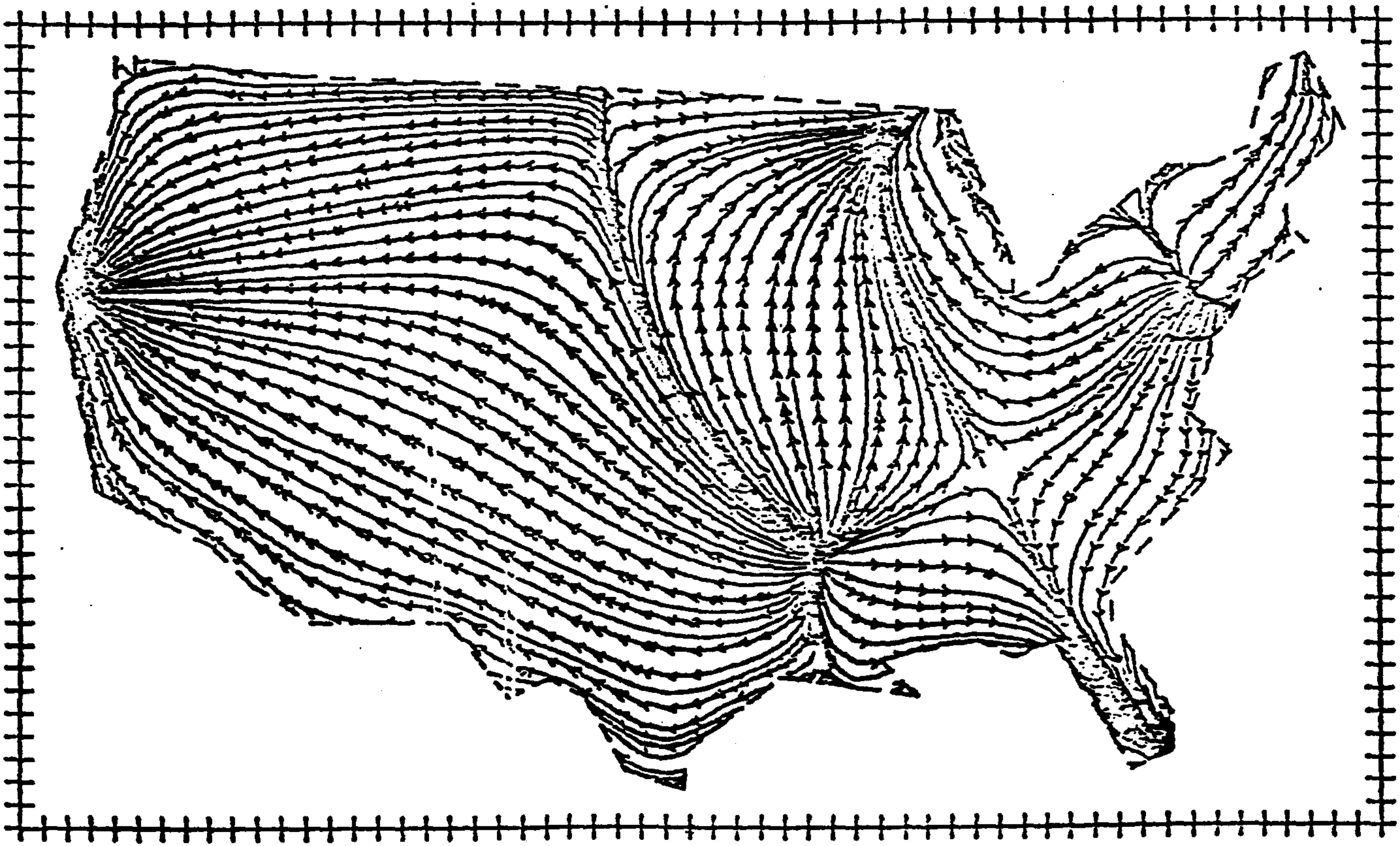
Solving the Equations Yields the Potential

Shown here by contours



The raster is indicated by the tick marks. The arrows are the gradients to the potentials. The streakline map is obtained by connecting the gradient vectors.

Dollar Bill Movement in the U.S.



The same technique can be applied to other types of movement

For example the migratory movement of people.

Population movement in the United States

Is recorded in migration tables using census collection units and published at several levels of resolution. The tables are of course asymmetric.

Average resolution is here defined by
(Area of domain/ Number of units)$^{1/2}$.

The resolution level is important because it tells one the size of pattern which can be detected.

Migration Data Often Come in the Form of Square Tables

The rows represent the “from” places and the columns the “to” places.

The tables are not symmetrical!

Nine Region Migration Table

US Census 1965-1970

(Note asymmetry. There are places of depletion and accumulation.)

	1	2	3	4	5	6	7	8	9
1 New England	—	180,048	79,223	26,887	198,144	17,995	35,563	30,528	110,792
2 Mid-Atlantic	283,049	—	300,345	67,280	718,673	55,094	93,434	87,987	268,458
3 East North Central	87,267	237,229	—	281,791	551,483	230,788	178,517	172,711	394,481
4 West North Central	28,977	60,681	286,580	—	143,860	49,892	185,618	181,868	274,629
5 South Atlantic	130,830	382,565	346,407	92,308	—	252,189	192,223	89,389	279,739
6 East South Central	21,434	53,772	287,340	49,828	316,650	—	141,679	27,409	87,938
7 West South Central	30,287	64,645	161,645	144,980	199,466	121,366	—	134,229	289,880
8 Mountain	21,450	43,749	97,808	113,683	89,806	25,574	158,006	—	437,255
9 Pacific	72,114	133,122	229,764	165,405	266,305	66,324	252,039	342,948	—

This is an example of a census migration table. There are also (50 by 50) state tables and county by county tables.

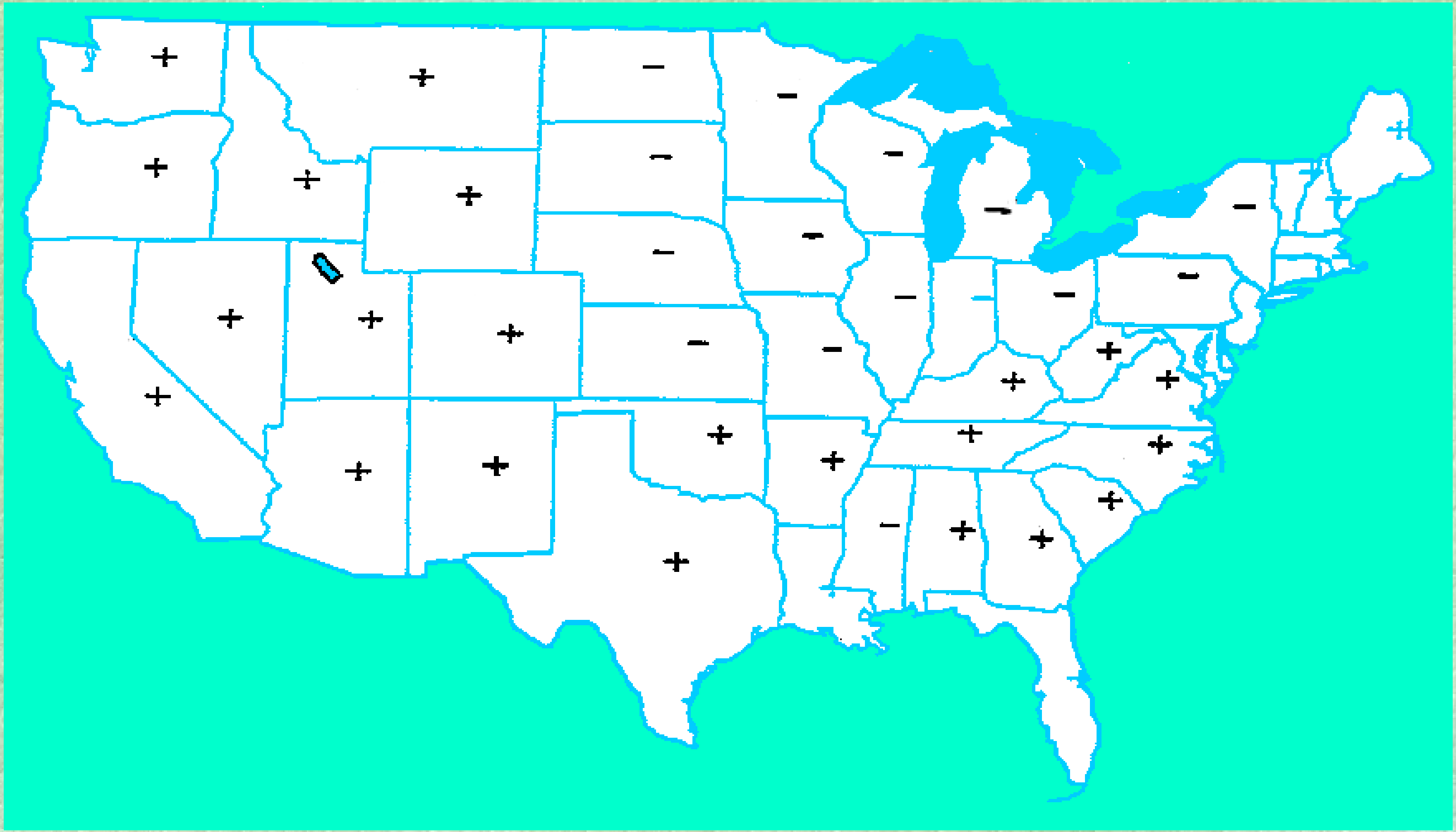
There is a great deal of spatial coherence in the migration pattern

In the US case the state boundaries hide the effect. Therefore a clearer picture emerges if they are omitted.

There is also temporal coherence.

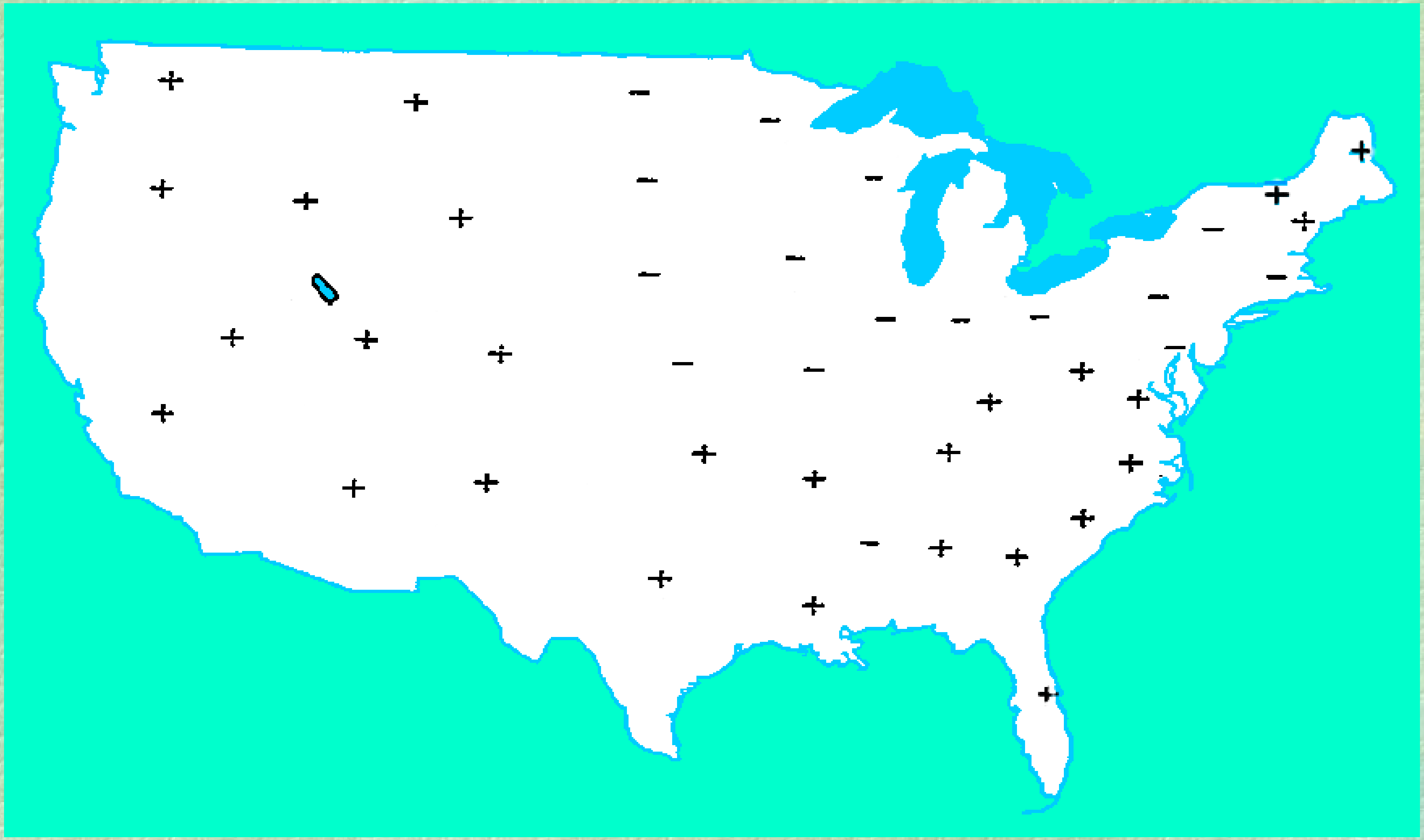
W. Tobler, 1995, “Migration: Ravenstein, Thornthwaite, and Beyond”, *Urban Geography,* 16(4):327-343.

The Population Change Information Can Be Positioned Locationally using centroids



Observe the spatial autocorrelation and how this is brought out more clearly by omitting the collection unit boundaries, as on the next map.

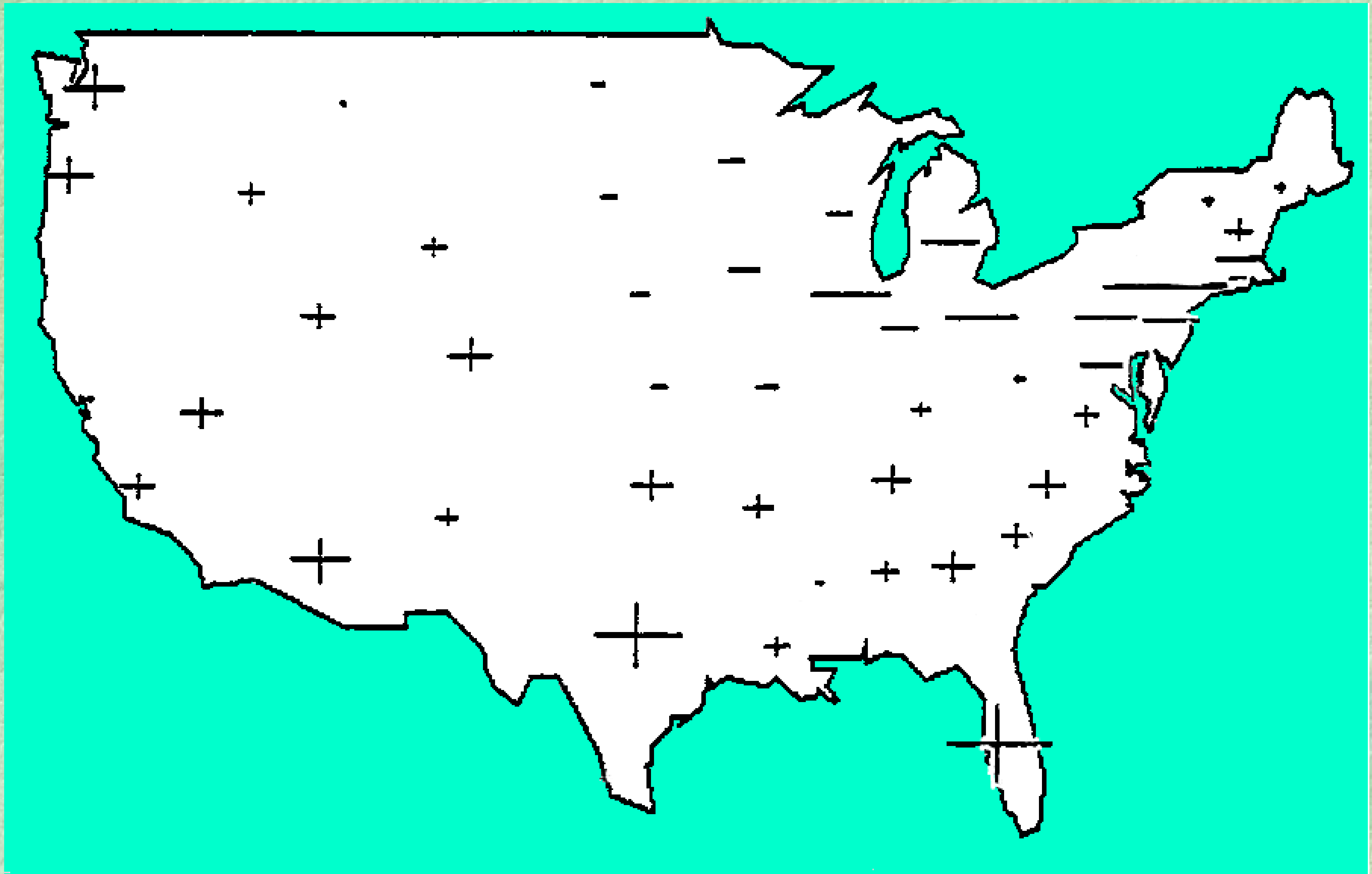
Population Change at State Centroids



The map is even better if the symbol size is made proportional to the magnitude of the change, as on the next map

Gaining and Losing States

Based on the marginals of a 48 by 48 migration table



Notice that only the Net Movements from the Table are being used

These are the difference of the marginals.

In-movement minus out-movement.

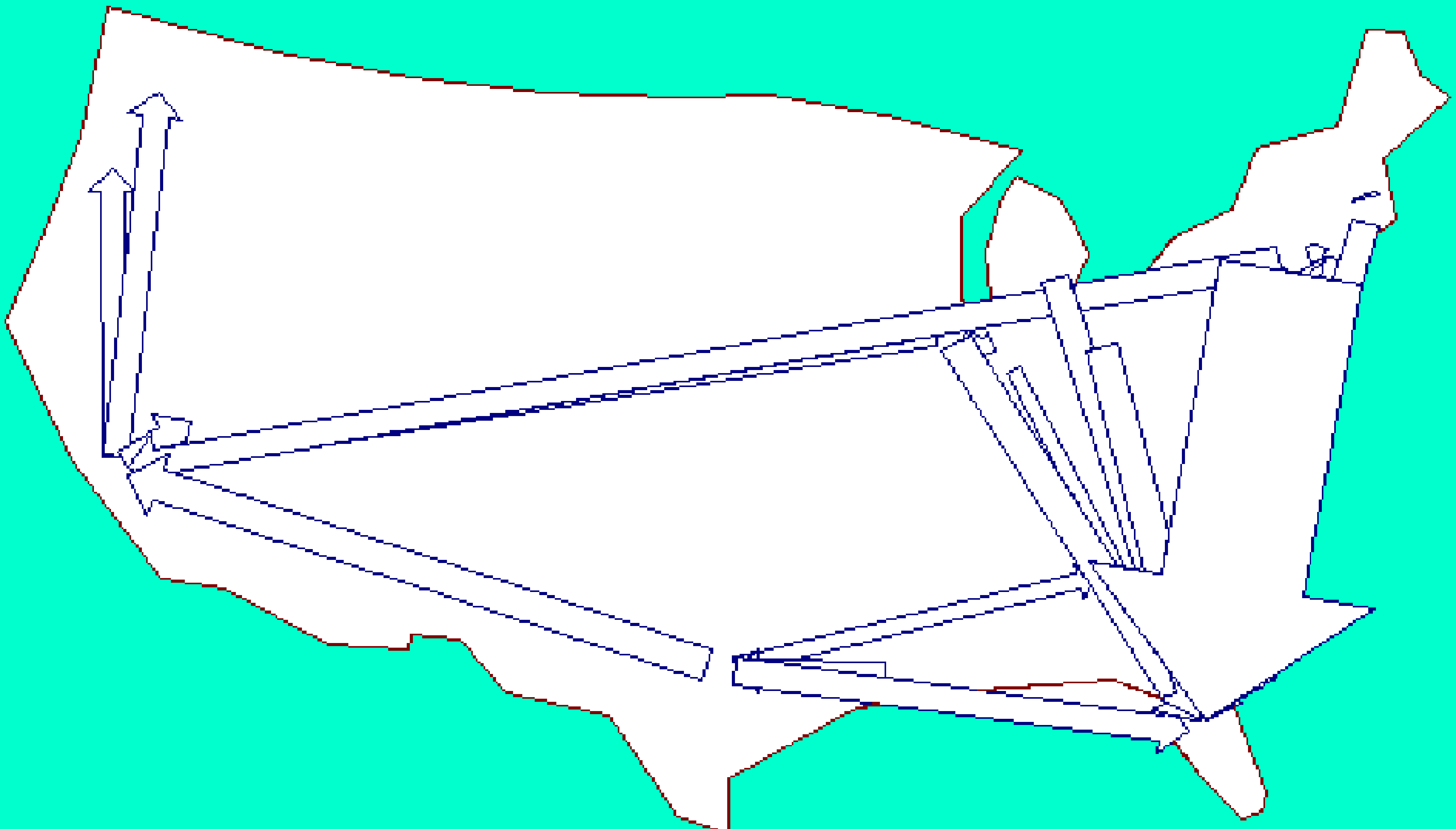
From the asymmetry of the table margins one can compute an attractivity, or pressure to move. Of course this requires a model.

G. Dorigo, & Tobler, W., 1983, “Push-Pull Migration Laws”, *Annals*, AAG, 7391):1-17.

The Conventional Net Movement Map

Based on movement between state centroids

(Computer sketch. Optimum deletion: values below mean ignored)



Using a model, this information can be converted to a potential field and its gradient

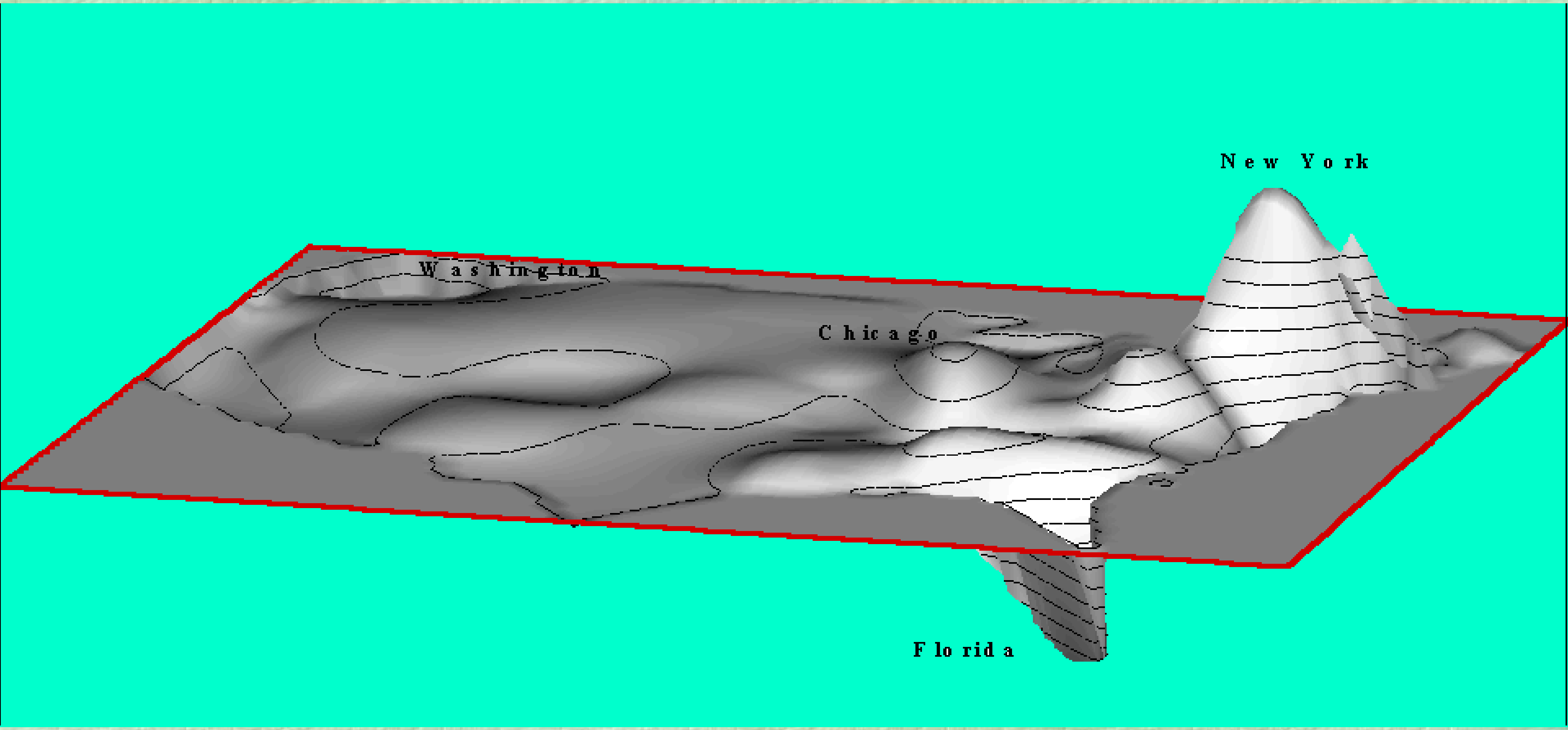
The model is, in essence, again a continuous version of the familiar gravity model.

The gradient vectors can also be connected to give a streakline map.

The next maps are based on the same observations as the previous map.

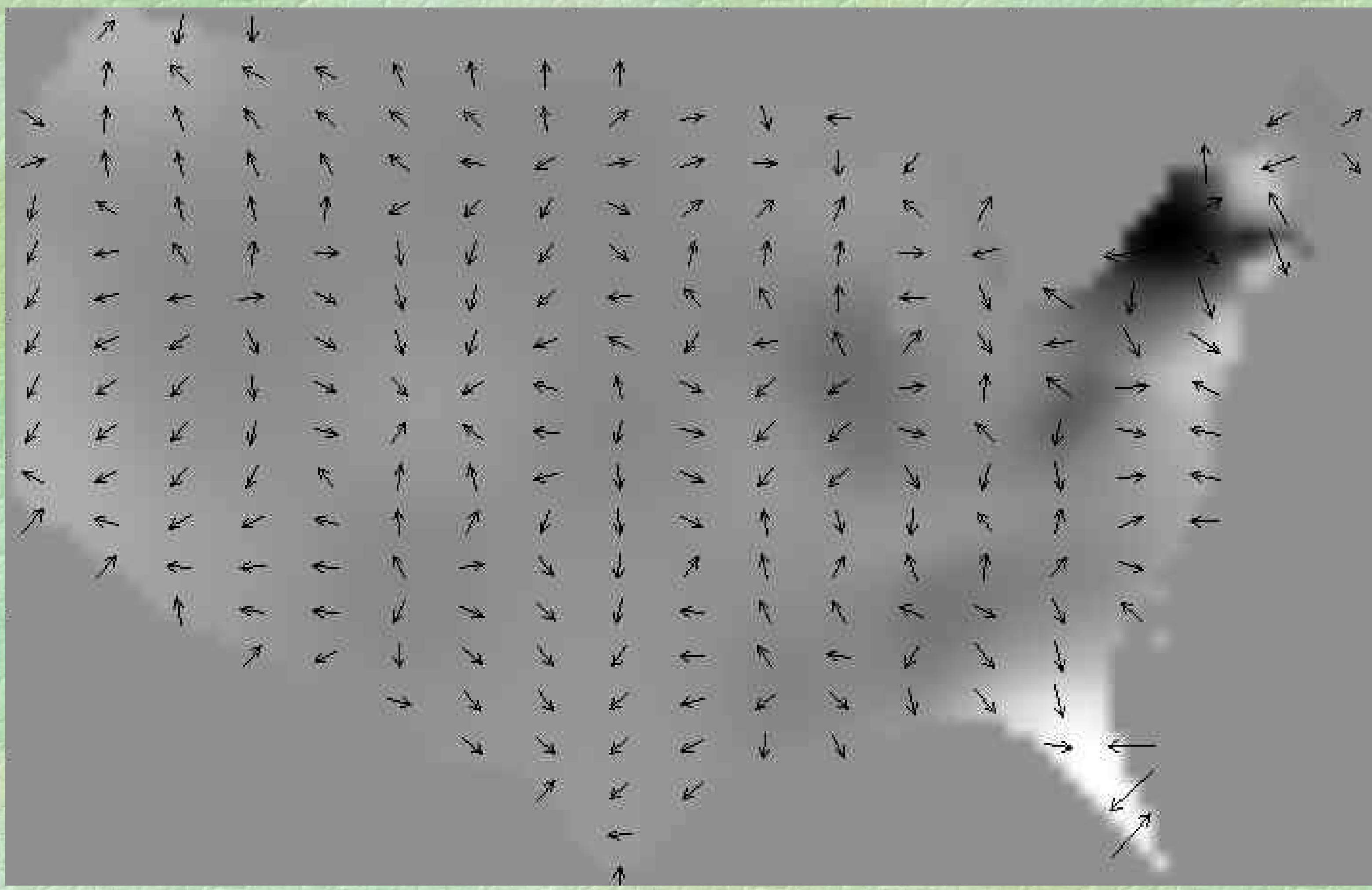
The Pressure to Move in the US

Based on a continuous spatial gravity model



Migration Potential and Gradients

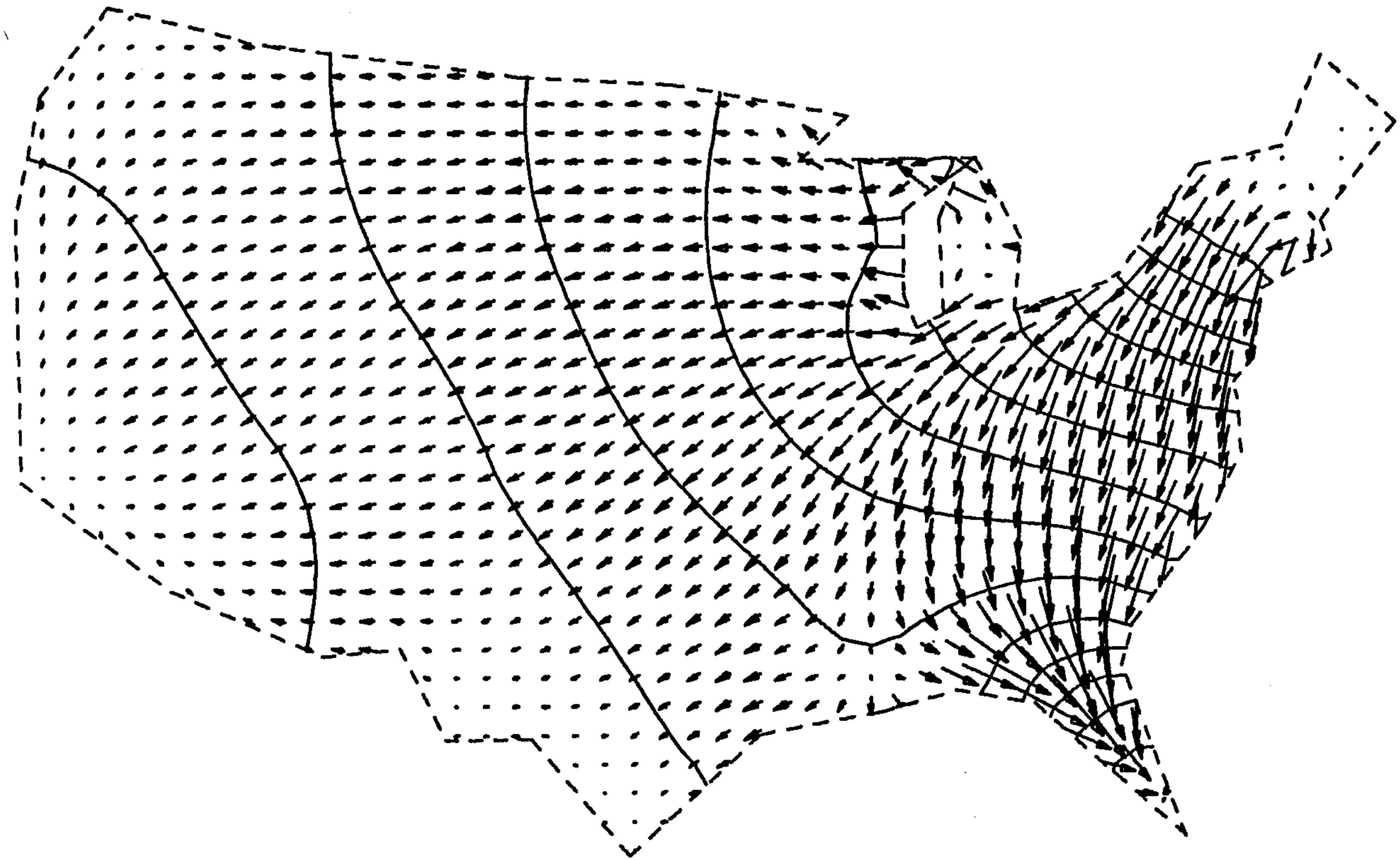
Another view of the same model



Another view

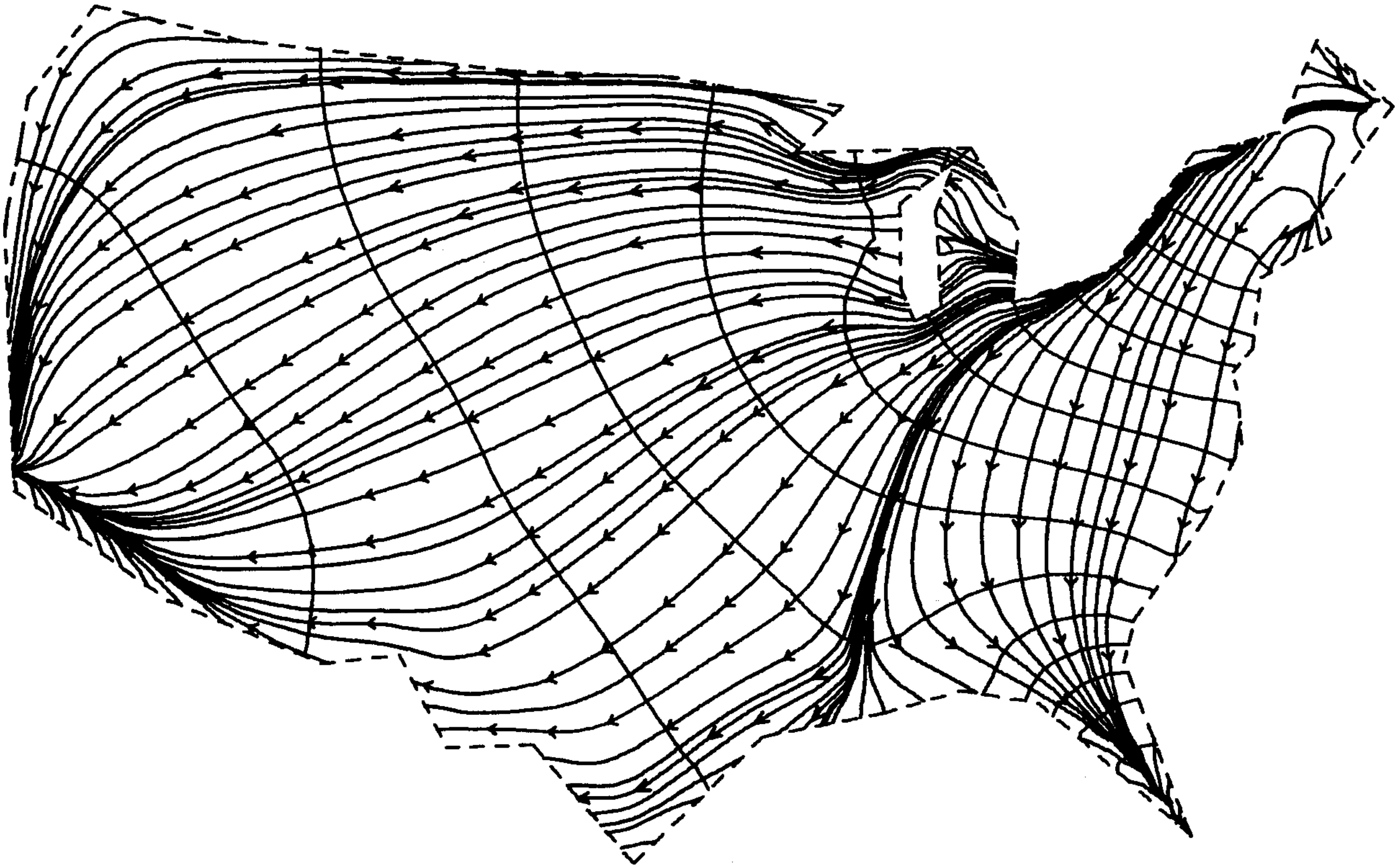
The Migration Potentials and Gradients

with the potentials shown as contours



Migration Potentials and Streaklines

with the gradient vectors connected to form streaklines

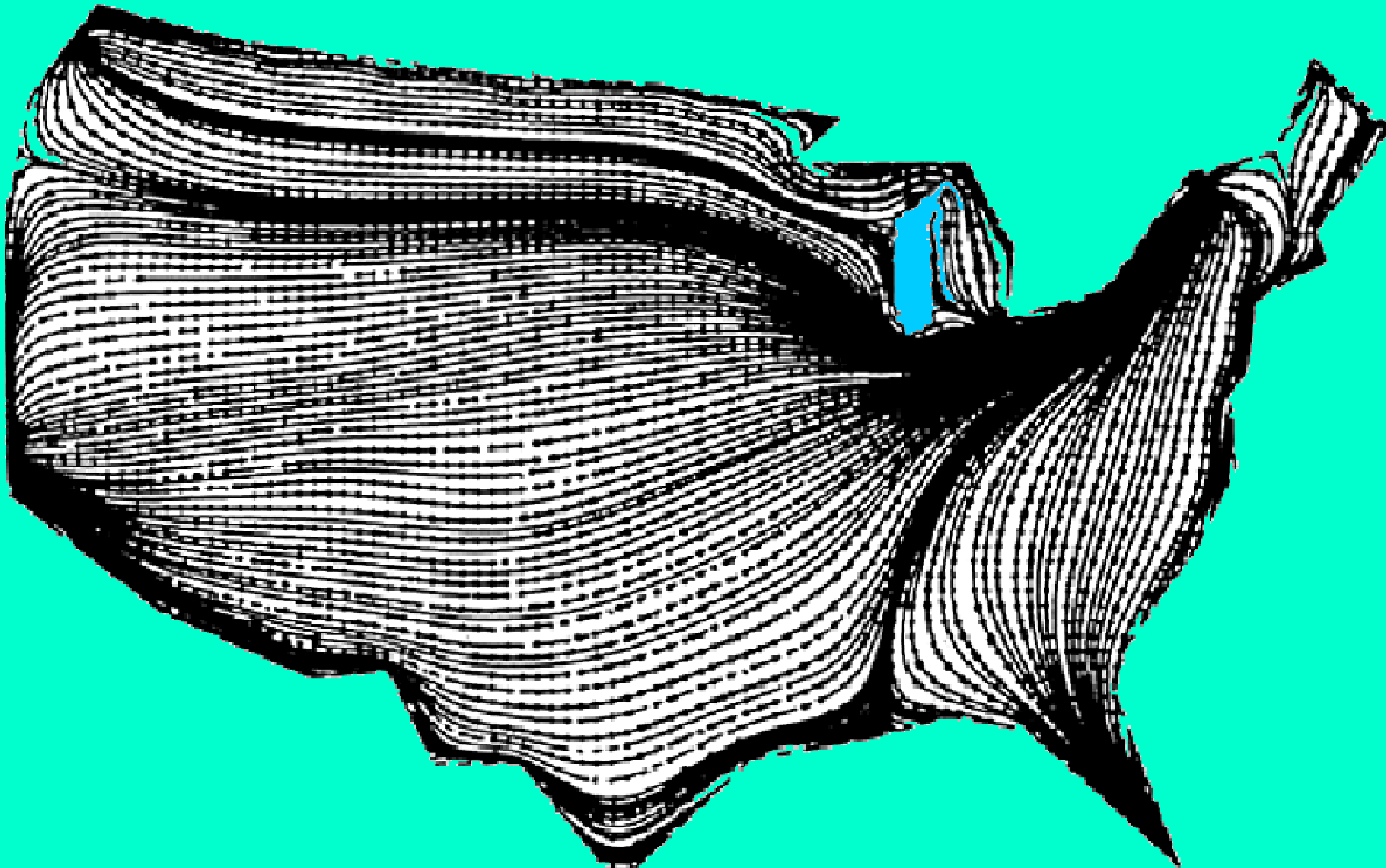


Recall that several million people migrate during the 5 year census period

The next map shows an ensemble average, not the path of any individual.

But observe, not unrealistically, that the people to the East of Detroit tend to go to the Southeast, and Minnesotans to the Northwest, and the remainder to the Southwest.

16 Million People Migrating



By the insertion of arbitrary areal boundaries, and by measuring the amount of flux across these boundaries, one can obtain information not contained in the original data, i.e., make a prediction.

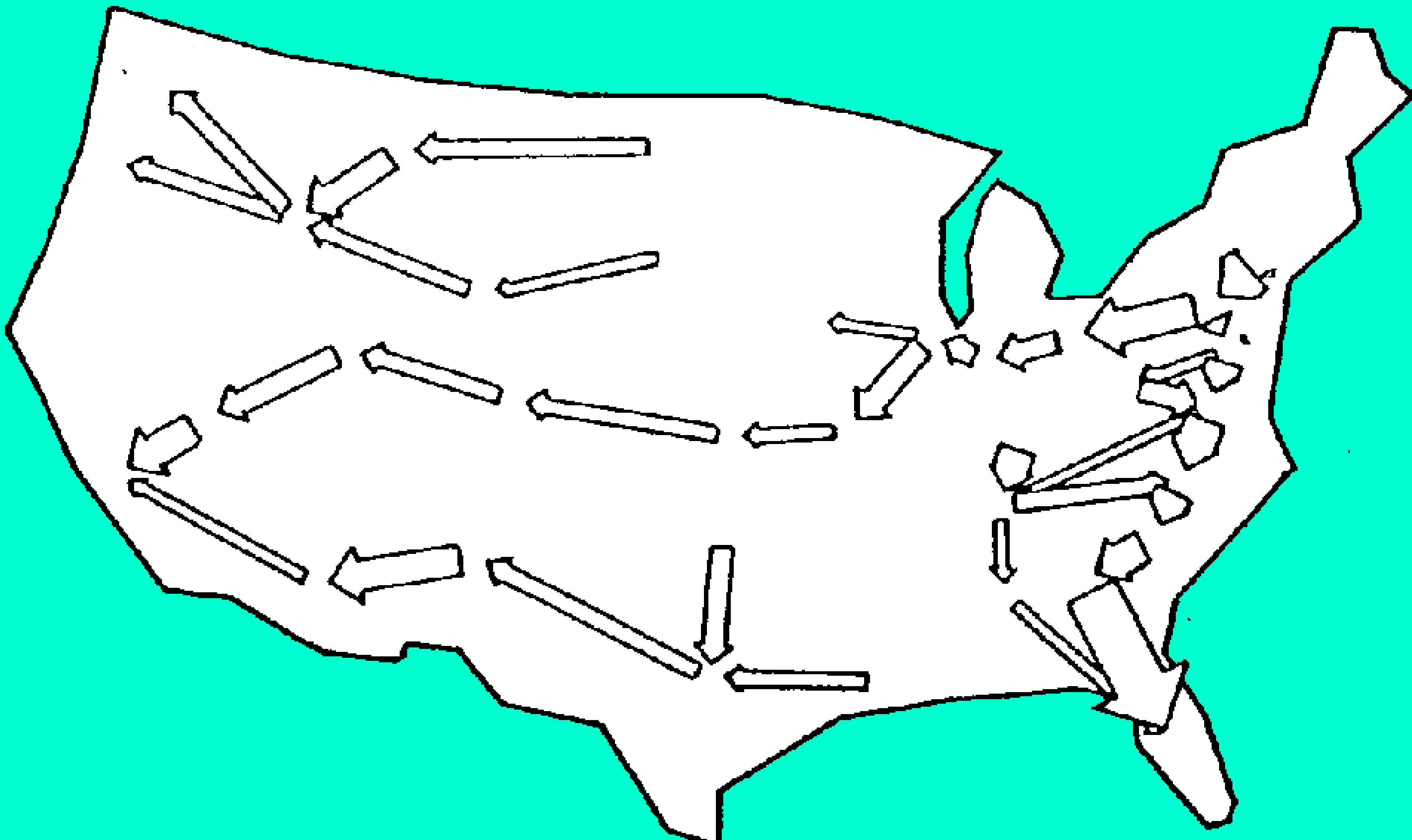
It’s like using a cookie cutter pressed into the continuous flow model to look at an arbitrary piece and computing the flow across its borders.

The next map is an example, using state boundaries.

The US Census Bureau does not provide this information. The model is used to make the prediction

Major Flux Across State Boundaries

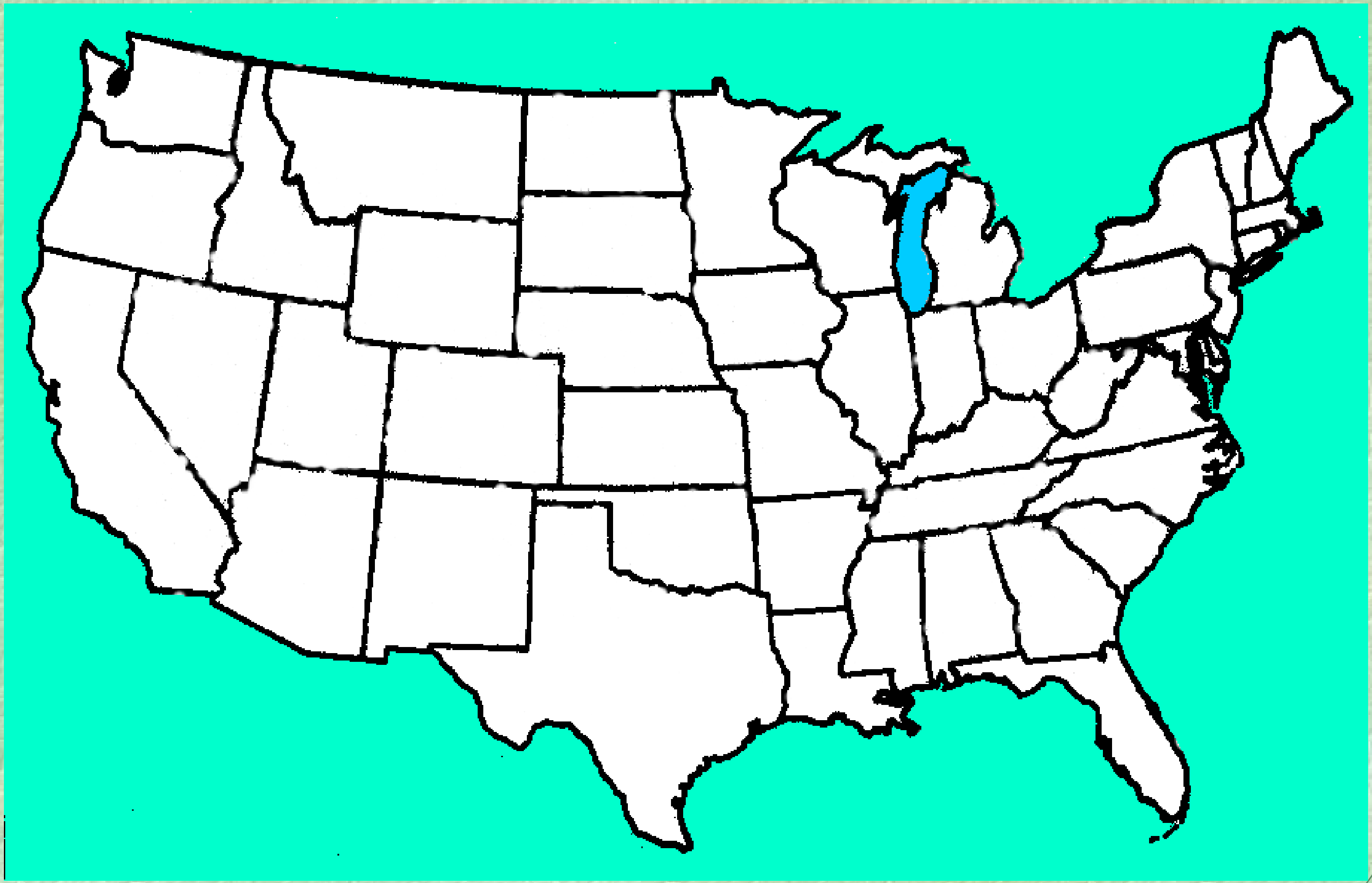
Predicted from the model and table marginals



That these migration maps resemble maps of wind or ocean currents is not surprising given that we in fact speak of migration flows and backwaters, and use many such hydrodynamic terms when discussing movement phenomena.

The Previous Maps Have Used Observations based on States

Patterns within urban areas could not be seen.



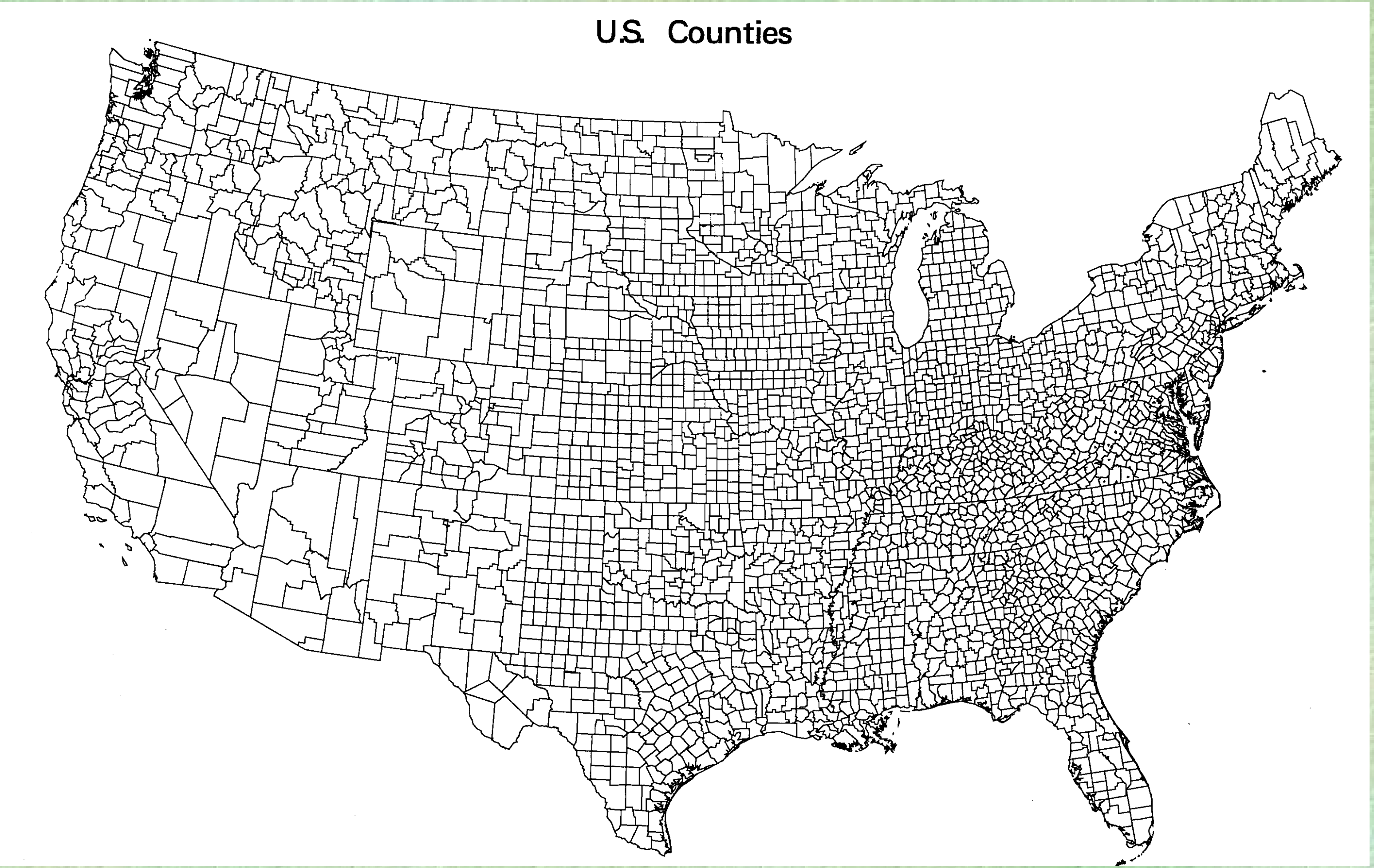
Average resolution 409 km. Detectable objects > 800 km in size.

If we used the 3,141 counties of the United States the migration table could contain 9,862,740 numbers

This is not a lot for a computer, but for humans?

We need models and visualization techniques!

County Units



Average resolution ~55 km. Patterns >110 km detectable.

Still not sufficient to see movement within cities.

The $9x10^6$ numbers in a county to county table could not be comprehended without some vizualization techniques or without a model.

Of course we know that most of the cells in the county to county table would be empty.

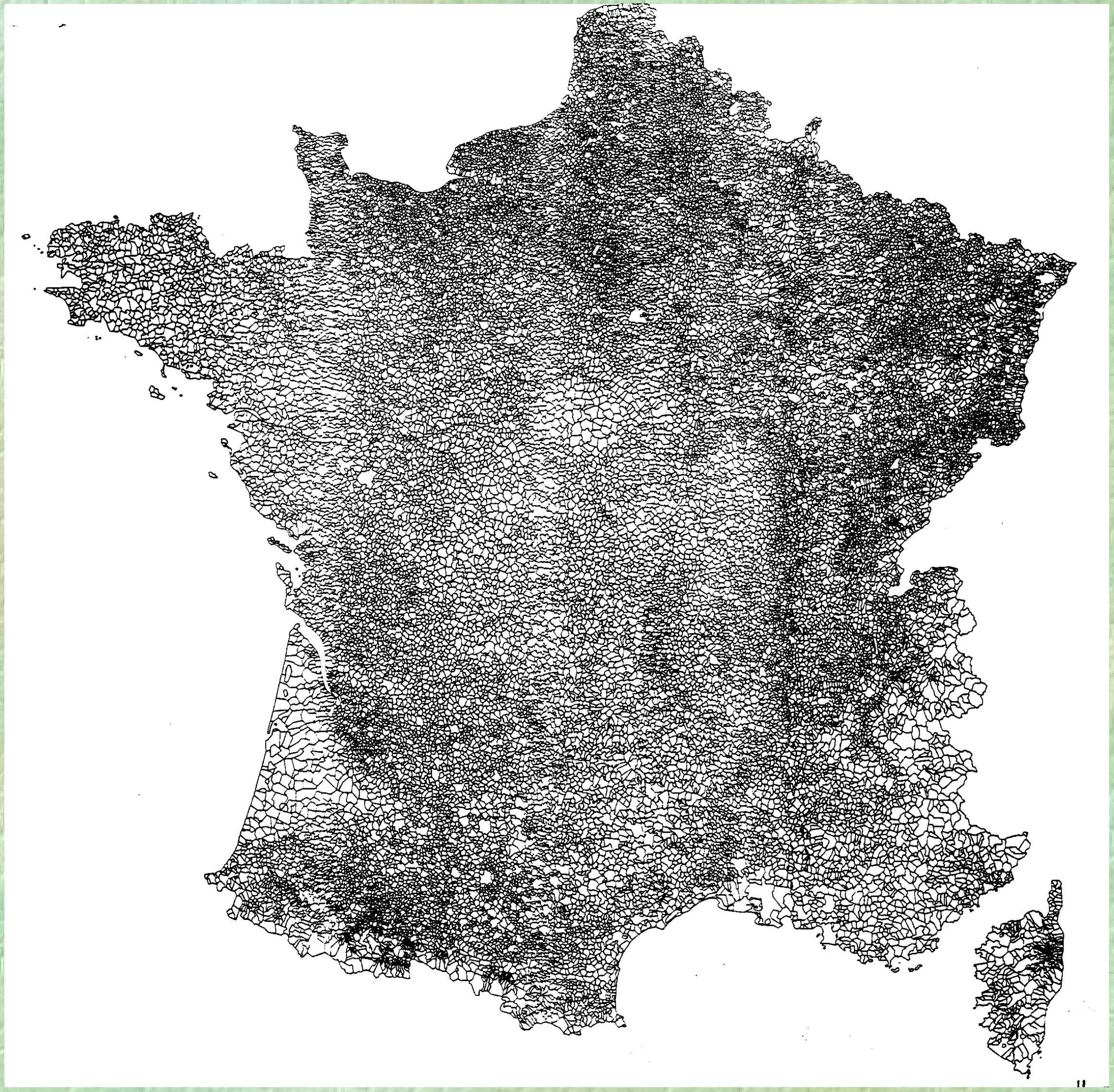
If the 3141 by 3141 US county migration table has only 5% of the cells with non-zero entries, that is still almost half a million numbers!

Think Big!

The 36,545 communes of France could yield a migration or interaction table with as many as 1,335,537,025 entries.

(3 km average resolution)

36,545 Communes of France



This many communes could lead to a migration table with 1,335,537,025 entries.

For a world table of

international migration
refugee movements
commodity trade

one would have a table of nearly 40,000 entries.

It is thus no surprise that few such tables exist.

Have you noticed that almost no statistical volumes contain from-to tables.

Migration in Switzerland

At several levels of resolution

(Guido Dorigo, University of Zürich)

Notice that one can pick out the Alps just from the migration pattern

and that it is, at the finest level of resolution, almost like turbulence.

Importantly we see that reducing the resolution has the effect of a high frequency spatial filter.

The spatial filtering effect is easily seen

by displaying vector fields at several levels of resolution.

The next several maps are of net migration in Switzerland.

3.6 km resolution (3090 Gemeinde)

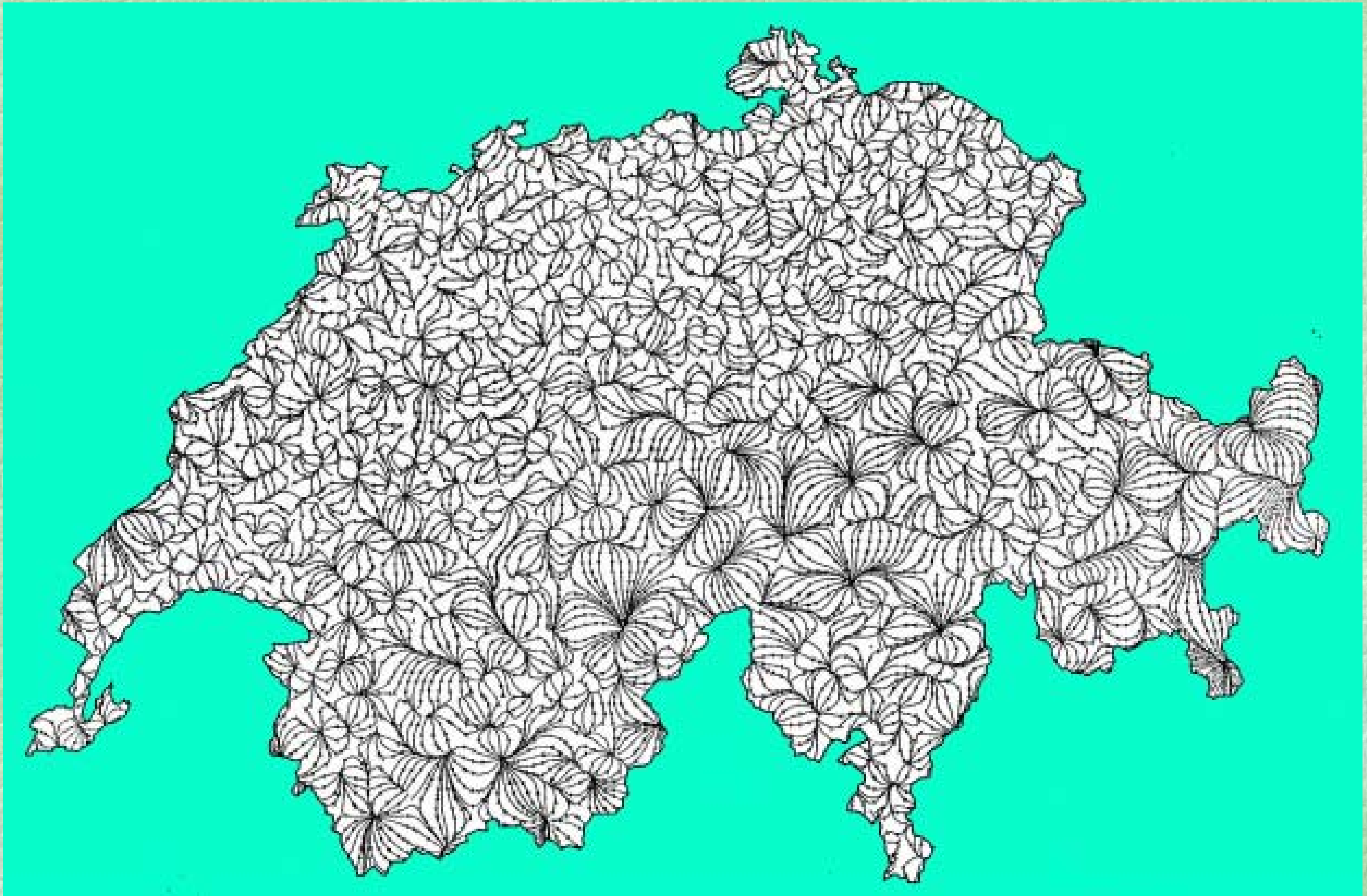
14.7 km resolution (184 Bezirke)

39.2 km resolution (26 Kantone)

Maps by Guido Dorigo, University of Z♒rich

Net Migration In Switzerland

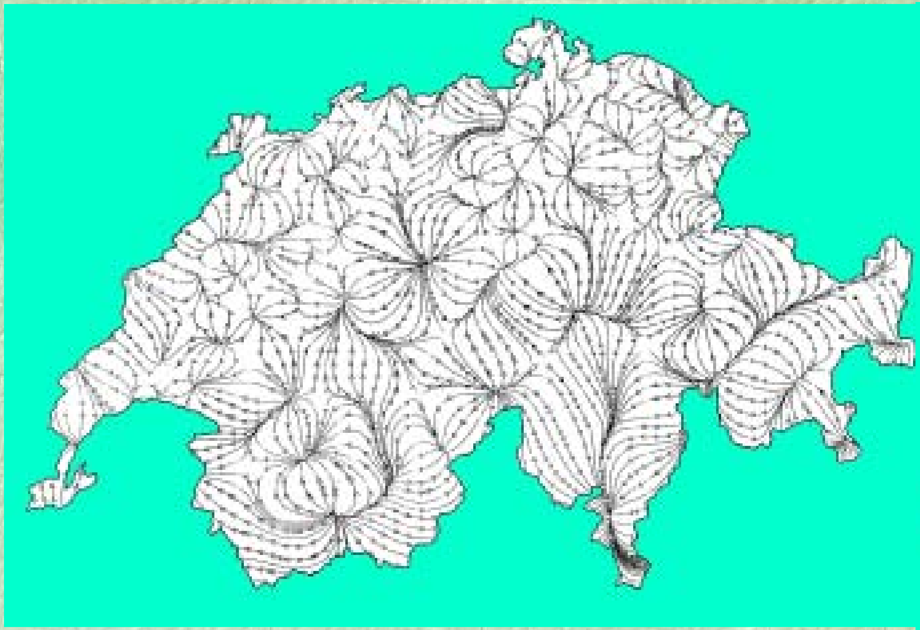
At 3.5 km resolution, 3090 Gemeinde



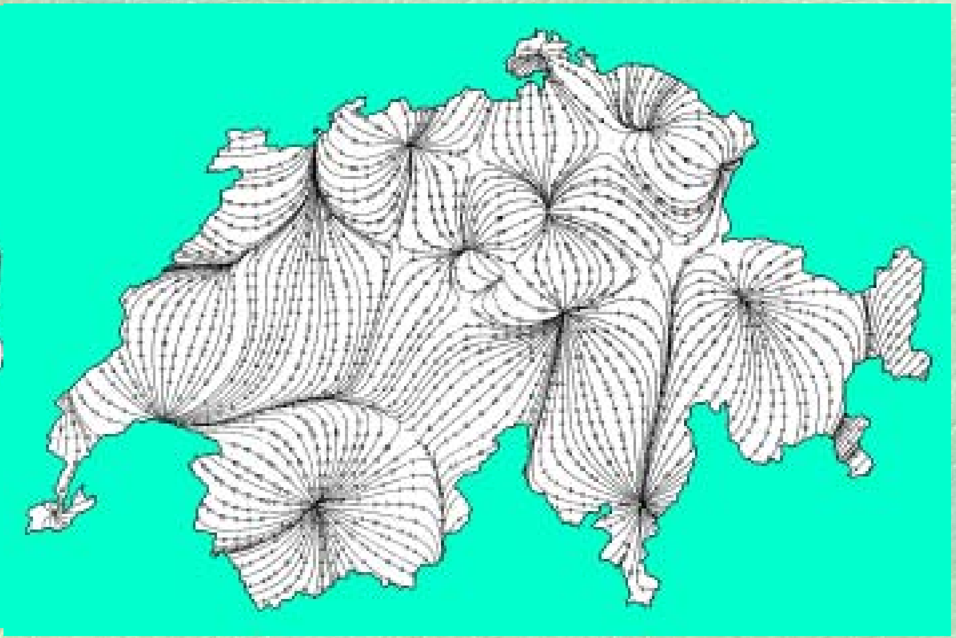
Swiss Migration at Reduced Resolution

To emphasize the filtering effect of resolution

14.7 km resolution, 184 Bezirke



39.2 km resolution, 26 Kantone



A table giving the interaction of everybody on earth with everyone else would be $6x10^9$ by $6x10^9$ in size, and that's only for one time interval!

But it is a very sparse table, each person having at most a few thousand connections.

In this presentation I have emphasized

Movement

As represented by various kinds of geographic modeling.

I appreciate your attention and thank you.

Waldo Tobler
Professor Emeritus
Geography department
University of California
Santa Barbara, CA 93106-4060
http://www.geog.ucsb.edu/people/tobler.htm