CSISS WORKSHOP

Introduction to Spatial Pattern Analysis in a GIS Environment

Measures of Spatial Pattern: Global and Local Statistics

Arthur Getis

Pattern Statistics

- GLOBAL

  I, c, K, G, Knox, Mantel, Tango, Grimson, Cuzick and Edwards, Kernels, Scan

- LOCAL

  I_i, c_i, G_i, G_i*, GWR, O_i

Global Statistics

- Nearest Neighbor
- K-Function
- Global Autocorrelation Statistics
  - Moran's I
  - Geary's c
  - Semivariance

Matrix Representation: **WY**

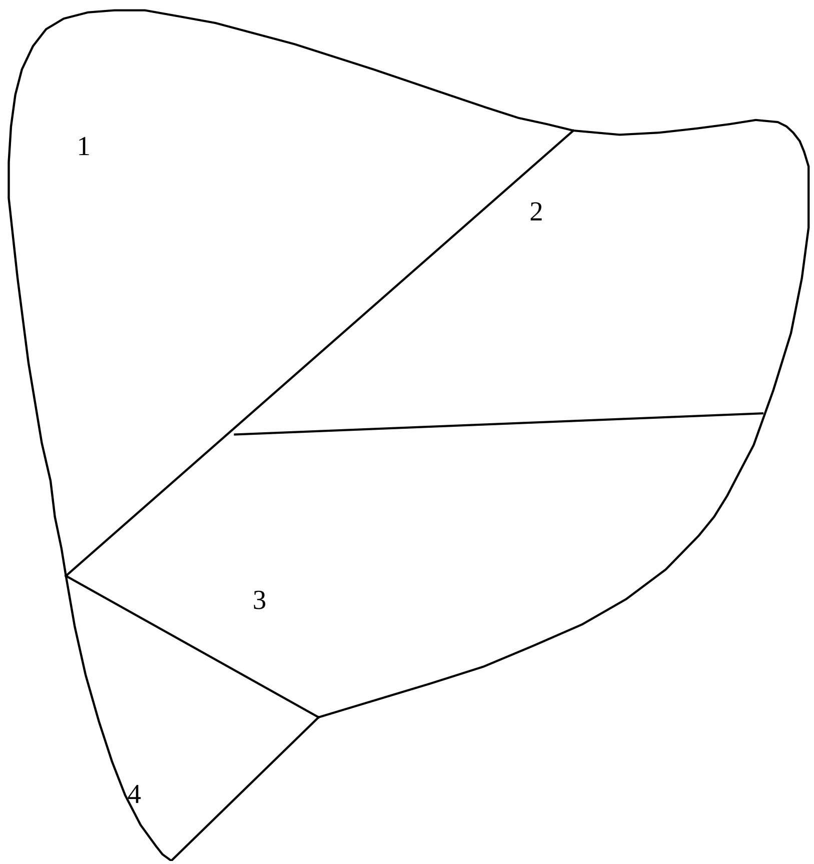
- **W**
- The Spatial Weights Matrix
- The Spatial Association of All Sites to All Other Sites
- d, d^2, 1/0, 1/d

- **Y**
- The Attribute Association Matrix
- The Association of the Attributes at Each Site to the Attributes at All Other Sites
- +,-,/,x

The Spatial Weights Matrix

W is the formal expression of the spatial association between objects

(it is the pair-wise geometry of objects being studied).



0	1	1	0
1	0	1	0
1	1	0	1
0	0	1	0

Typical W

- Spatially contiguous neighbors (rook, queen: one/zero)
- Inverse distances raised to a power: ($1/d$, $1/d^2$, $1/d^5$)
- Geostatistics functions (spherical, gaussian, exponential)
- Lengths of shared borders (perimeters)
- All centroids within distance *d*
- n^{th} nearest neighbor distance
- Links (number of)

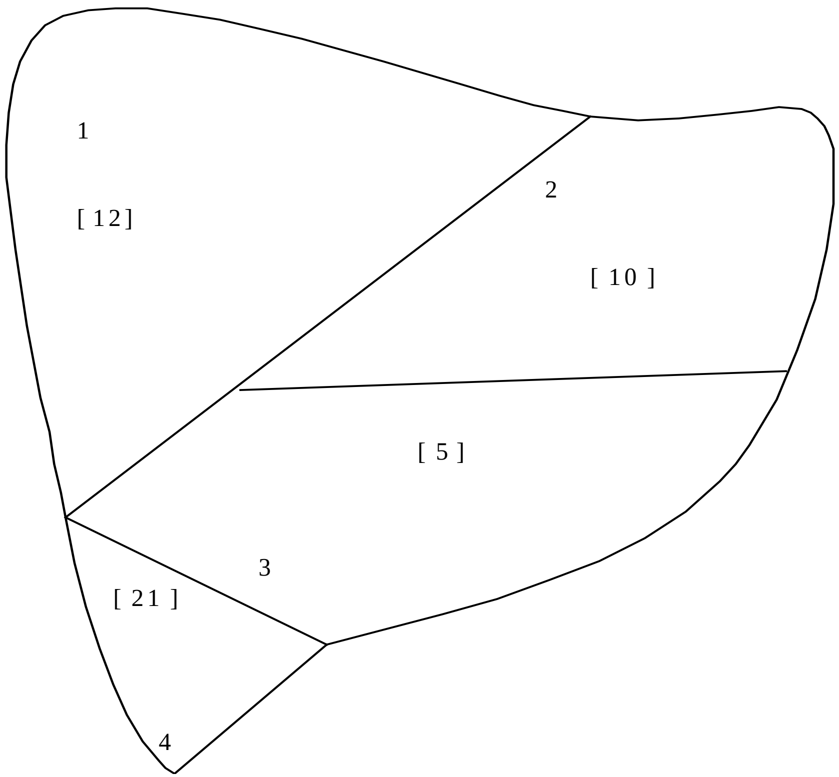
The Attribute Matrix

Y

The variable under study. One variable at a time. Interval scale (other scales under special conditions).

For example, residuals from regression;

a socio-economic variable (number of crimes, household income, number of artifacts, etc.)



0	2	7	-9
-2	0	5	-11
-7	-5	0	-16
9	11	16	0

Attribute Relationships

Y

- **Types of Relationships**

Additive association (clustering): $(Y_i + Y_j)$

Multiplicative association (product): $(Y_i Y_j)$

Covariation (correlation): $(Y_i - Ybar)(Y_j - Ybar)$

Differences (homogeneity/heterogeneity): $(Y_i - Y_j)$

Inverse (relativity): (Y_i/Y_j)

- All Relationships Subject to Mathematical Manipulation (power, logs, abs, etc.)

WY: Covariance

- Set **W** to preferred spatial weights matrix
- (rooks, queens, distance decline, etc.)
- Set **Y** to $(x_i - \mu)\ (x_j - \mu)$
- Set scale to $n/W\ \Sigma(x_i - \mu)^2$
- $\mathrm{I} = n\ \Sigma\,\Sigma\ W_{ij}\ (x_i - \mu)\ (x_j - \mu)\ /\ W\ \Sigma(x_i - \mu)^2$

where W is sum of all W_{ij} *and* $i \neq j$

This is Morans's I.

WY: Additive

- Set **W** to 1/0 spatial weights matrix
- 1 within *d*; 0 outside of *d*
- Set **Y** to $(x_i + x_j)$
- Set scale to $\Sigma W_{ij}(d) / \Sigma (x_i)$
- $G(d) = \Sigma W_{ij}(d)\ (x_i + x_j) / \Sigma (x_i)$ *and i ♑ j*

This is Getis and Ord's G.

WY: Difference

- Set **W** to preferred spatial weights matrix
- Set **Y** to $(x_i - x_j)^2$
- Set scale to $(n\text{-}1)/2W\Sigma(x_i - \mu)^2$
- c = $(n - 1)\ \Sigma\ \Sigma\ W_{ij}\ (x_i - y_i)^2 /\ 2W\Sigma(x_i - \mu)^2$

where W is sum of all W_{ij} *and i ♑ j*

This is Geary's c.

WY: Difference

- Set **W** to 1/0 weights matrix; 1 within *ah* and 0 otherwise; *a* is an integer; *h* is a constant distance
- Set **Y** to $(x_i - x_j)^2$
- Set scale to *1/2*
- $\chi(ah) = 1/2 \Sigma \Sigma W_{ij} (x_i - x_{j)})^2$

This is the semi-variogram.

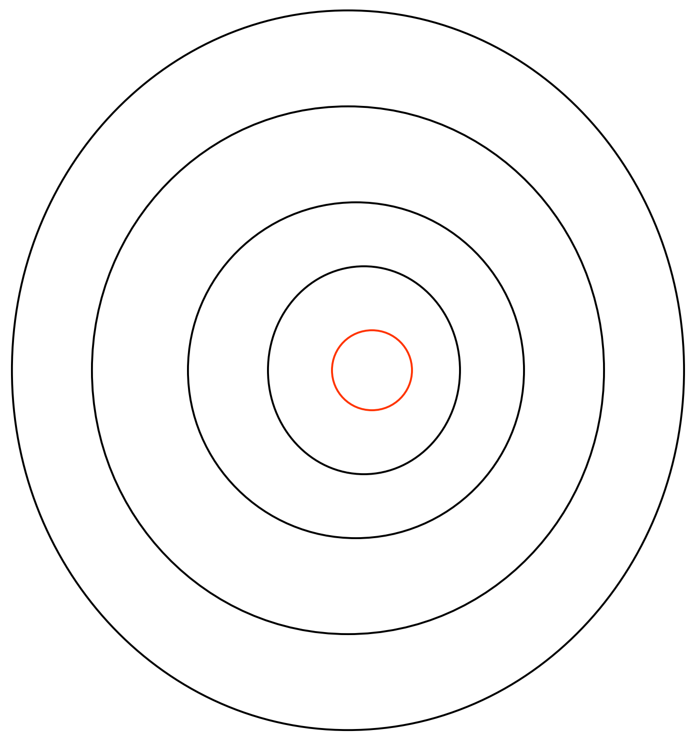
Local Statistics

- Global Statistics reworked for focussing on *i*
- LISA statistics (Local Indicators of Spatial Association)

Moran's I_i , Geary's c_i

- Clustering Statistics

Getis and Ord's G_i and G_i*



The Getis-Ord Approach

$$G_i{*}(d) = [\Sigma_j\, w_{ij}{}^{*}(d)\, x_j]/\Sigma_j x_j$$

- Normally distributed
- Tests for statistical significance

The G_i^* Statistic

- The G_i^* statistic is local, that is, it is focused on sites and is normally distributed. It is designed to yield a measure of pattern in standard normal variates.
- Indicates the extent to which a location (site) is surrounded to a distance d by a cluster of high or low values.
- The input is a file containing coordinates for each house and, for example, the Y variable. The user specifies maximum search distance and number of increments.
- The output file contains a listing of the $G_i^*(d)$ value for sample point at a specified distance (d).

The Critical Distance

- The G_i* values are computed around each observation as distance increases.
- When the absolute values fail to rise, the cluster diameter is reached. This is the critical distance d_c.
- Spatial association weakens beyond d_c.

Example Ranges

(SA = Santa Ana)